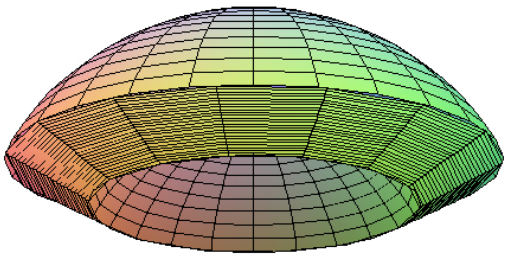
Math 241: Quiz 11

Show ALL Work Name **Solutions**

(1) Let G be the solid inside the cone $z = \sqrt{(x^2+y^2)/3}$ and between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$ (as shown). Express the volume of G as a triple integral in spherical coordinates ρ, θ and ϕ with appropriate limits of integration. Do NOT evaluate the triple integral.

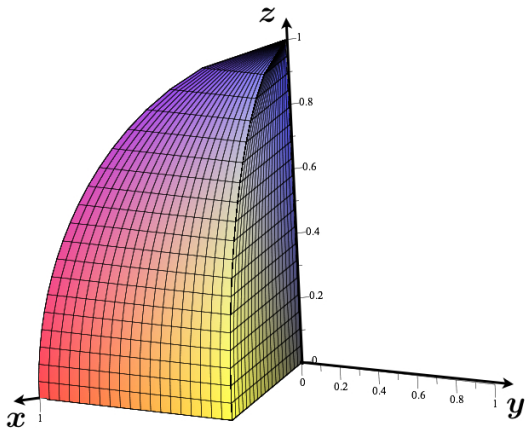


Triple integral in spherical coordinates:

$$\boxed{\int_0^{2\pi}\int_0^{\pi/3}\int_2^3 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta}$$

Note that the cone $z = \sqrt{(x^2+y^2)/3}$ can be written as $z = r/\sqrt{3}$. Switching to spherical coordinates, this becomes $\rho\cos\phi = (\rho\sin\phi)/\sqrt{3}$ or, equivalently, $\tan\phi = \sqrt{3}$. So the cone is simply $\phi = \pi/3$. Inside the solid, the angle ϕ goes from 0 (on the positive z-axis) to $\pi/3$ (on the cone).

(2) Fill in the six boxes below to correctly complete interchanging the order of integration. The picture to the right depicts the solid that is to be used for the limits of integration, but the first triple integral below should be used to obtain specific information about the solid.



$$\int_0^1 \int_y^1 \int_0^{\sqrt{1-x^2}} f(x,y,z)\, dz\, dx\, dy$$

$$= \int_{\boxed{0}}^{\boxed{1}} \int_{\boxed{0}}^{\boxed{\sqrt{1-z^2}}} \int_{\boxed{y}}^{\boxed{\sqrt{1-z^2}}} f(x,y,z)\, dx\, dy\, dz$$