Answers to Math 241 Final, Spring 2001

Part I:

(1) (a) $\langle 1, -4, 5\rangle$
(b) -1
(c) $\langle 2, 3, 2\rangle$

(2) (a) $-57/5$
(b) $\langle 3, 12\rangle$ or $\langle 1, 4\rangle$ or $\langle 1/\sqrt{17}, 4/\sqrt{17}\rangle$

(3) $(0, 1, 0)$
$(0, -1, 0)$

(4) 0

(5) Maximum $1/4$
Minimum $-1/4$

(6) $36\pi\sqrt{3}/5$

(7) (a) saddle
(b) not a critical point
(c) local mininimum

Part II:

(1) $-1/2$

(2) (a) $(2 - t - s)^2 + (-4 - t - s)^2 + (3 - 3s)^2$
(b) $t = -2$ and $s = 1$
(c) $(3, 1, 2)$ and $(0, 4, 2)$

(3) (a) $61\pi/3$
(b) $32/5$

(4) (a) Let $\vec{n_1} = \langle 1, -2, -1\rangle$ and $\vec{n_2} = \langle a, b, c\rangle$. Then $\vec{n_1}$ is perpendicular to $\mathcal{P}$ and $\vec{n_2}$ is perpendicular to $ax + by + cz = d$. The angle θ between $\vec{n_1}$ and $\vec{n_2}$ is either 60° or 120°. Deduce the given equation from $\vec{n_1} \cdot \vec{n_2} = |\vec{n_1}||\vec{n_2}| \cos\theta$.
(b) Use that $a^2 + b^2 + c^2 = 6$ in the equation in part (a).
(c) Suppose $ax+by+cz = d$ is $\mathcal{P}'$ or $\mathcal{P}''$ so that Q and R are on $ax+by+cz = d$. Then $\vec{QR} = \langle 1, 1, -1\rangle$ is perpendicular to $\vec{n_2}$ in part (a). From $\vec{n_2} \cdot \vec{QR} = 0$, deduce $a + b - c = 0$.
(d) $(-1, -1, -2)$ and $(2, -1, 1)$
(e) $x + y + 2z = 9$ and $2x - y + z = 12$