MATH 141: TEST 3

Name __________________________________

Instructions and Point Values: Put your name in the space provided above. Check that your test has exactly 6 different pages including one blank page. Work each problem below and show ALL of your work. You do not need to simplify your answers unless a problem indicates otherwise. Do NOT use a calculator.

Problem (1) is worth 10 points.
Problem (2) is worth 24 points.
Problem (3) is worth 12 points.
Problem (4) is worth 14 points.
Problem (5) is worth 12 points.
Problem (6) is worth 14 points.
Problem (7) is worth 14 points.

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(1) Given that $\int_0^5 f(x)\,dx = 7$, $\int_0^4 f(x)\,dx = 2$, and $\int_3^5 f(x)\,dx = -4$, calculate $\int_0^3 f(x)\,dx$.

(2) Calculate each of the following integrals.

(a) $\displaystyle\int (x-1)(x+1)\,dx$

(b) $\displaystyle\int_0^{\pi/2} \sin(2t)\,dt$

(c) $\displaystyle\int_0^1 x(1-x)^9\,dx$

(d) $\displaystyle\int \frac{(1+\sqrt{x})^{100}}{\sqrt{x}}\,dx$

(3) Calculate $\sum_{n=1}^{100}\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right)$. Simplify your answer.

Answer: $\boxed{}$

(4) Find the solution to the differential equation $3y^2y' + 4x = 0$ given the initial condition that $y = 1$ when $x = 1$. Put your answer in the form $y = f(x)$.

Answer: $\boxed{}$

(5) Given that $F(x) = \int_x^{x^2} \sqrt{1+t^2}\, dt$, calculate $F(1)$, $F'(x)$, and $F'(1)$. (Show work.)

$F(1) = \boxed{}$

$F'(x) = \boxed{}$

$F'(1) = \boxed{}$

(6) Calculate the area of the region bounded by the graphs of $y = x^2$ and $y = x$.

(7) Calculate the integral $\int_a^b f(x)\,dx$ boxed below in the following way. Divide the interval $[a,b]$ into n equal subintervals, calculate the area of the corresponding circumscribed polygon, and then let $n \to \infty$. You should make use of the formula

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

Your final answer should be a number.

$$\boxed{\int_0^3 (4x+3)\,dx}$$