MATH 141: TEST 3

Name ______________________________

Instructions and Point Values: Put your name in the space provided above. Check that your test has exactly 6 different pages including one blank page. Work each problem below and show <u>ALL</u> of your work. You do not need to simplify your answers. Do <u>NOT</u> use a calculator.

Problem (1) is worth 12 points.
Problem (2) is worth 20 points.
Problem (3) is worth 12 points.
Problem (4) is worth 14 points.
Problem (5) is worth 14 points.
Problem (6) is worth 14 points.
Problem (7) is worth 14 points.

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(1) Given that $\int_0^3 f(x)\,dx = 1$, $\int_0^4 f(x)\,dx = 6$, and $\int_1^4 f(x)\,dx = 2$, calculate $\int_1^3 f(x)\,dx$.

(2) Calculate each of the following integrals.

(a) $\displaystyle\int_0^{\pi/2} \sin\theta \, \cos\theta \, d\theta$

(b) $\displaystyle\int \frac{(t+1)^2}{t} - \frac{1}{t} \, dt$

(c) $\displaystyle\int \sqrt{x+1}\, dx$

(d) $\displaystyle\int x\sqrt{x+1}\, dx$

(3) Given that $F(x) = \int_1^x \sqrt{t^4+1}\,dt$, calculate $F(1)$, $F'(1)$, and $F''(1)$. Justify your answers.

$F(1) = \boxed{}$

$F'(1) = \boxed{}$

$F''(1) = \boxed{}$

(4) Find all functions $f(x)$ for which $f'(x) = (f(x))^2$. (This is a differential equation problem. It may help you to replace $f(x)$ with y.)

(5) Calculate the area of the region bounded by the graphs of $x = y^2 - y$ and $x = y - y^2$.

(6) The region in the first quadrant bounded above by the ellipse $x^2 + 2y^2 = 9$ and below by the line $y = 2x$ revolves about the $x-$axis to form a solid. Calculate the volume of the solid.

(7) Calculate the integral $\int_a^b f(x)\,dx$ boxed below in the following way. Divide the interval $[a,b]$ into n equal subintervals, calculate the area of the corresponding circumscribed polygon, and then let $n \to \infty$. You should make use of the formula

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

Your final answer should be a number.

$$\boxed{\int_1^3 (3x-1)\,dx}$$