MATH 141: TEST 2

Name _____

Instructions and Point Values: Put your name in the space provided above. Check that you have 6 (different) test pages. Work each problem below and, unless I have indicated that it is not necessary, show ALL of your work. Do NOT use a calculator.

Problem (1) is worth 16 points.
Problem (2) is worth 18 points.
Problem (3) is worth 14 points.
Problem (4) is worth 34 points.
Problem (5) is worth 18 points.

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(1) Calculate each of the limits. You do not need to show work for parts (a), (b), and (c). You must show work for part (d).

(a) $\lim_{x\to\infty} \dfrac{4x+5}{3x-1} =$ $\boxed{}$ (b) $\lim_{x\to 1^+} \dfrac{x+1}{x-1} =$ $\boxed{}$ (c) $\lim_{x\to 2^-} \dfrac{x}{\sqrt{4-x^2}} =$ $\boxed{}$

(d) $\lim_{x\to\infty} \left(\sqrt{x^2+5x+4} - \sqrt{x^2-x}\right) =$ $\boxed{}$ (SHOW WORK!!)

(2) For each of the following, calculate the absolute maximum value and the absolute minimum value for $f(x)$ on the interval I. (I am more interested in your work than in the answers, so be sure to justify your answers.)

(a) $f(x) = 2x^3 + 3x^2 - 12x - 6$ and $I = [0, 2]$

Absolute Maximum Value: $\boxed{}$

Absolute Minimum Value: $\boxed{}$

(b) $f(x) = \sin x + \cos x$ and $I = [0, \pi]$

Absolute Maximum Value: $\boxed{}$

Absolute Minimum Value: $\boxed{}$

(3) (a) Use that $f(x+\Delta x) \approx f(x) + (\Delta x) f'(x)$ to explain why $(16.096)^{1/4} \approx 2.003$. Fill in the boxes below (but this is not sufficient for an explanation).

$f(x) =$ $\square$

$x =$ $\square$

$\Delta x =$ $\square$

(b) Using the Mean Value Theorem, explain why $(16.096)^{1/4} \leq 2.003$.

(4) For this page and the next page, $f(x) = \dfrac{(x-1)(x+1)}{x^2+3}$. You may use the following information concerning $f(x)$:

$$f'(x) = \frac{8x}{(x^2+3)^2} \quad \text{and} \quad f''(x) = \frac{-24(x-1)(x+1)}{(x^2+3)^3}.$$

Answer each part below. You may answer "Nowhere" or "None".

(a) What are the critical points for $f(x)$?

(b) On what intervals is $f(x)$ increasing?

(c) On what intervals is $f(x)$ decreasing?

(d) What are the local maximum values of $f(x)$?

(e) What are the local minimum values of $f(x)$?

(f) On what intervals is $f(x)$ concave up?

(g) On what intervals is $f(x)$ concave down?

(h) What are the inflection points of $f(x)$? (Give the x and y coordinates.)

(i) There is a horizontal asymptote. Write down its equation.

(j) Draw the graph of $y = f(x)$.

(5) Jill's boat (with her in it) is sinking 4 miles from the nearest point P on a straight shoreline. Jill swims to shore and jogs to the nearest town 8 miles down the shoreline from P. If she swims at a rate of 3 miles per hour and jogs at a rate of 5 miles per hour, how many miles from P should she come to shore if she wishes to reach the town as quickly as possible? Justify that your answer minimizes the time traveled.

Answer: | miles from P |