MATH 141: TEST 1

Name ______________________________

Instructions and Point Values: Put your name in the space provided above. Work each problem below and show ALL of your work. You do not need to simplify your answers. Do NOT use a calculator.

Problem (1) is worth 9 points.
Problem (2) is worth 9 points.
Problem (3) is worth 16 points.
Problem (4) is worth 24 points.
Problem (5) is worth 12 points.
Problem (6) is worth 16 points.
Problem (7) is worth 14 points.

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(1) Evaluate $\displaystyle\lim_{x\to 1} \frac{x^2 - 2x + 1}{x^3 - x^2 - x + 1}$.

(2) Evaluate $\displaystyle\lim_{x\to 0} \frac{(\cos x)\sin(2x)}{(\sin x)\cos(2x)}$.

(3) Let

$$f(x) = \begin{cases} x^2 & \text{if } x > 0 \\ 1 & \text{if } x = 0 \\ -x^2 & \text{if } x < 0. \end{cases}$$

Answer each of the following. If a limit does not exist, simply write, "The limit does not exist." Otherwise determine its value. If you draw an appropriate graph of $y = f(x)$, you may answer part (a) and (b), but <u>NOT</u> (c) or (d), without showing any further work.

(a) Evaluate $\lim_{x \to 0+} f(x)$.

(b) Evaluate $\lim_{x \to 0^-} f(x)$.

(c) Evaluate $\lim_{x \to 0} f(x)$. Explain

(d) Is $f(x)$ continuous at 0? Explain.

(4) Calculate each of the derivatives below, and put your answers in the appropriate boxes. You do not need to show any work on this page.

(a) $\frac{d}{dx}\left(\cos(x^2)\right)^{24} = \boxed{}$

(b) $\frac{d}{dt}\frac{1}{\sqrt{t^3 - 4t}} = \boxed{}$

(c) if $f(x) = (x+1)^{10}\sqrt{x}$, then $f'(x) = \boxed{}$

(d) $\frac{d}{dx}\left(\frac{\sin x}{x}\right) = \boxed{}$

(5) Find the equation of the tangent line to $y = x(x+1)^4$ at the point $(-2, -2)$.

(6) Using

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

calculate $f'(x)$ for

$$f(x) = \frac{1}{x^2 + 1}.$$

(You must use the definition of the derivative given above to receive credit for the problem.)

(7) Superman decides to fly around the world. He leaves the Daily Planet, where he works, and observes that the distance he has travelled after t seconds is $t^3 + 3t$ miles. How long does it take before he is travelling 15 miles per second? (That's over 50000 miles per hour, but who cares.)

Answer: | seconds