MATH 141 PRACTICE PROBLEMS FOR TEST 3

1. Calculate the following:

(a) $\int 0\,dx$

(b) $\int_1^4 \frac{dt}{\sqrt{t}}$

(c) $\int_0^2 (x-2)(3x+1)\,dx$

(d) $\int \sqrt{x}(x^2+1)\,dx$

(e) $\int_0^{\pi/2} \sin(x/2)\,dx$

(f) $\int_0^1 (t+1)^{7/2}t\,dt$

2. Calculate $\sum_{k=1}^{100}\left(\frac{1}{k(k+2)}\right)$.

3. Given that $\int_0^2 f(x)\,dx = 3$, $\int_1^3 f(x)\,dx = 5$, and $\int_0^3 f(x)\,dx = 6$, calculate $\int_0^1 f(x)\,dx$, $\int_1^2 f(x)\,dx$, and $\int_3^2 f(x)\,dx$.

4. Given that $F(x) = \int_2^x \sqrt{t^3+1}\,dt$, calculate $F(2)$, $F'(2)$, and $F''(x)$.

5. Suppose $y' - y^2x = 0$. Also, suppose that $y = 4$ when $x = 0$. Here $y = f(x)$. What is $f(x)$?

6. Calculate the area of the region bounded by the graphs of $x = y^2 - y$ and $y = x$.

7. Let $f(x)$ be a continuous function on an interval $[a, b]$ with $a < b$. Suppose that

$$\int_a^b (f(x))^2\,dx \leq \int_a^b f(x)\,dx.$$

Does it necessarily follow that $f(x) \leq 1$ for at least one value of x in $[a, b]$? Explain your answer.

8. Calculate the integral $\int_a^b f(x)\,dx$ boxed below in the following way. Divide the interval $[a, b]$ into n equal subintervals, calculate the area of the corresponding circumscribed polygon, and then let $n \to \infty$. You should make use of the formulas

$$(*) \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \quad \text{and} \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

Your final answer should be a number.

$$\boxed{\int_0^3 (3x^2 + 2x + 1)\,dx}$$

9. Prove the first formula given in $(*)$ above.

10. The region in the first quadrant bounded by $y = 4$ and $y = x^2$ is rotated about the x-axis to form a solid. What is the volume of the solid?