MATH 141: PRACTICE PROBLEMS FOR TEST 2

(1) Observe that 5 is a pretty good approximation to $\sqrt{25.03}$. Using that $f(x+h) \approx f(x) + hf'(x)$, explain how one can obtain the approximation 5.003 to $\sqrt{25.03}$. What should x and h be?

(2) (a) Explain why the Mean Value Theorem implies $\sqrt{25.03} \leq 5.003$.
(b) Explain why the Mean Value Theorem implies $\sqrt{25.03} \geq 5 + (0.03/10.006)$.

(3) Define $f(x) = 2\sin x - x$ for $0 \leq x \leq \pi$.
(a) Find the critical points of $f(x)$.
(b) Where on $[0, \pi]$ is $f(x)$ concave up?
(c) What is the absolute maximum value of $f(x)$?
(d) What is the absolute minimum value of $f(x)$?

(4) A string 28 inches long is divided into 2 pieces. One piece is used to form a square, and the other piece is used to form a rectangle with its length 3 times its width. If the sum of the areas of the square and rectangle is maximized, what are the lengths of the 2 pieces of string? (Allow for the possibility that a piece has length 0.)

(5) Let $f(x) = (x^2+4)/(x+2)^2$. Your teacher is nice and gives you that

$$f'(x) = \frac{4(x-2)}{(x+2)^3} \quad \text{and} \quad f''(x) = \frac{-8(x-4)}{(x+2)^4}.$$

(a) What are the critical points of $f(x)$?
(b) What are the horizontal asymptotes of $f(x)$?
(c) What are the vertical asymptotes of $f(x)$?
(d) What are its local maximum values?
(e) What are its local minimum values?
(f) What are its inflection points?
(g) Graph $y = f(x)$.

(6) Calculate $\lim_{x\to\infty} \left(\sqrt{x^3+4x} - x^{3/2}\right)$.

(7) If $y' = \sqrt{x}/y$, then what are the possible values of y?