MATH 141: FINAL EXAM

Name ________________________________

Instructions and Point Values: Put your name in the space provided above. Check that your test contains 14 different pages including one blank page. Work each problem below and show ALL of your work. Unless stated otherwise, you do not need to simplify your answers. You should NOT use Calculus material in your answers that are unrelated to this course. Do NOT use a calculator.

There are 300 total points possible on this exam. There are 2 parts. The first part consists of 20 problems each worth 10 points. The second part consists of 5 problems each worth 20 points.

PART I. Calculate each of the following. An asterisk ($*$) next to a problem number indicates that you do not need to show work for that problem. You should show work for every other problem.

(1) The equation of the tangent line to the graph of $y = x^3 + x + 1$ at the point $(1, 3)$. (You can use any material from this course.)

(2) $\displaystyle\lim_{x\to 0} \frac{x^2}{(\cos x) - 1}$

(3) $\lim_{x \to 1^-} \dfrac{|x| - 1}{x - 1}$ (Be Careful.)

(4) $\lim_{x \to 2} \dfrac{x^2 - 4}{x^2 - 3x + 2}$

(5) $\lim_{x \to \infty} \left(\sqrt{x^2 + 4x} - \sqrt{x^2 - 3x + 1} \right)$

(6) The derivative of $x^2 + 3$ by using the definition of the derivative, namely

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

(To receive credit for the problem make sure you use from this course only the definition of the derivative and information about limits.)

(7)* $\dfrac{d}{dt}\left(t\sqrt{t+1}\right)$

(8)* $\dfrac{d}{dx}\left(\dfrac{\sqrt{x}}{\sin x}\right)$

(9)* $f'(x)$ if $f(x) = \cos\left(\sqrt{x^2+1}\right)$

(10) $f'(2)$ given that $f(x) = \sin\left(g(x^2-1)\right)$ where $g(x)$ is a function with $g(3) = \pi$ and with derivative $g'(x) = 2^x + 3^{x-1}$

(11) $y = f(x)$ if $y' + xy^2 + y^2 = 0$ and if $f(0) = 1$

(12) How fast two planes are separating at 1:00 P.M. if one plane, flying east at 200 miles per hour, goes over a certain town at 11:00 A.M. and the second plane at the same altitude, flying north at 300 miles per hour, goes over the same town at noon. (Simplify Your Answer!!)

(13)* $\dfrac{d}{dx}\left(\displaystyle\int_{x^2}^{\sqrt{x}} \sqrt{t^2+1}\,dt\right)$

(14) $\displaystyle\int \sqrt{t}\,(t-1)\;dt$

(15) $\displaystyle\int_0^{\sqrt{\pi}} x\cos\left(x^2\right)\, dx$

(16) $\displaystyle\int \frac{\sin\theta}{(1+\cos\theta)^2}\, d\theta$

(17) $\displaystyle\int x(2x-1)^{5/2}\, dx$

(18) $\displaystyle\int_0^{\pi^2} \left(\frac{d}{dt}(\cos\sqrt{t})\right)\, dt$

(19) $\displaystyle\int_1^4 f(x)\,dx$ given that $\displaystyle\int_2^3 f(x)\,dx = 1$, $\displaystyle\int_2^4 f(x)\,dx = 6$, and $\displaystyle\int_1^3 f(x)\,dx = 2$

(20) The area of the region bounded by the graphs of $x = y^2 - 1$ and $y = x - 1$.

PART II. Answer each of the following. Make sure your work is clear. If you do not know how to answer a problem, tell me what you know that you think is relevant to the problem. If you end up with an answer that you think is incorrect, tell me this as well. Better yet, tell me why you think it is incorrect. In other words, let me know what you know.

(1) Calculate the integral $\int_a^b f(x)\,dx$ boxed below in the following way. Divide the interval $[a,b]$ into n equal subintervals, calculate the area of the corresponding circumscribed polygon, and then let $n \to \infty$. You should make use of the formula

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}.$$

Your final answer should be a number.

$$\boxed{\int_0^4 (4x+1)\,dx}$$

(2) For this page and the next page, $f(x) = \dfrac{x^2 - 1}{\sqrt{x^2 + 1}}$. The following information is given to you (you may use it):

$$f'(x) = \frac{x(x^2 + 3)}{(x^2 + 1)^{3/2}} \quad \text{and} \quad f''(x) = \frac{-3(x - 1)(x + 1)}{(x^2 + 1)^{5/2}}.$$

(a) Find all the critical points for $y = f(x)$.

(b) Find all local maximum values of $f(x)$.

(c) Find all local minimum values of $f(x)$.

(d) Find all asymptotes for the graph of $y = f(x)$.

(e) On what intervals is the graph of $y = f(x)$ concave up?

(f) Find all the inflection points for $y = f(x)$.

(g) Graph $y = f(x)$.

(3) The region bounded by the graphs of $y^2 = 4 - x$, the line $y = 0$, and the line $x = 0$ in the first quadrant is revolved about the line $x = 4$ to form a solid. Calculate the volume of this solid.

(4) What point on the graph of $y = \sqrt{x^2 - 2x + 2}$ is closest to the point $(1, -2)$? Justify your answer. There is a way of answering this question without using Calculus, and I don't mind if you do it that way; however, if you don't use Calculus, you should be careful to explain precisely what you are doing.

(5) Explain why

$$\int_0^1 \frac{1}{\left(1+\sqrt{t}\right)^3}\,dt = \frac{1}{4}.$$

In other words, calculate the integral and simplify your answer to obtain the value $1/4$.