MATH 122: TEST 4 REVIEW

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Office Hours on Tuesday: 10:00-10:45 a.m.
1:30-2:30 p.m.

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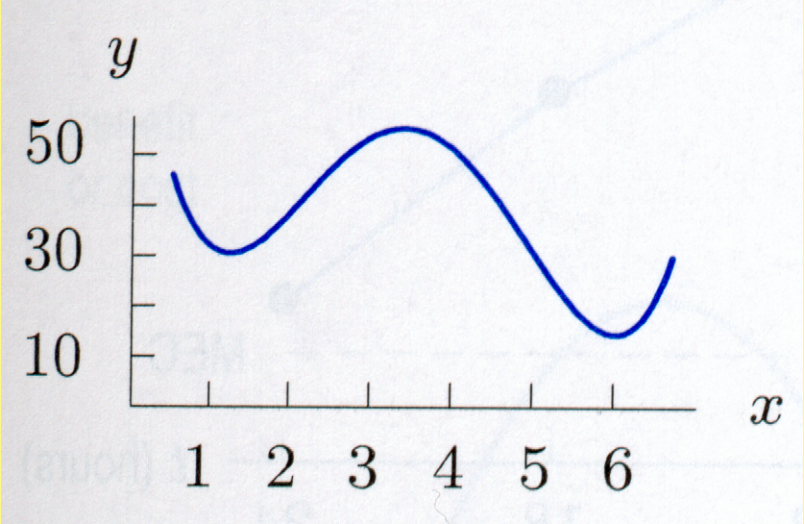
Office Hours on Tuesday: 10:00-10:45 a.m.
1:30-2:30 p.m.
electronic hours too

Pages 213-216

Problem 1:



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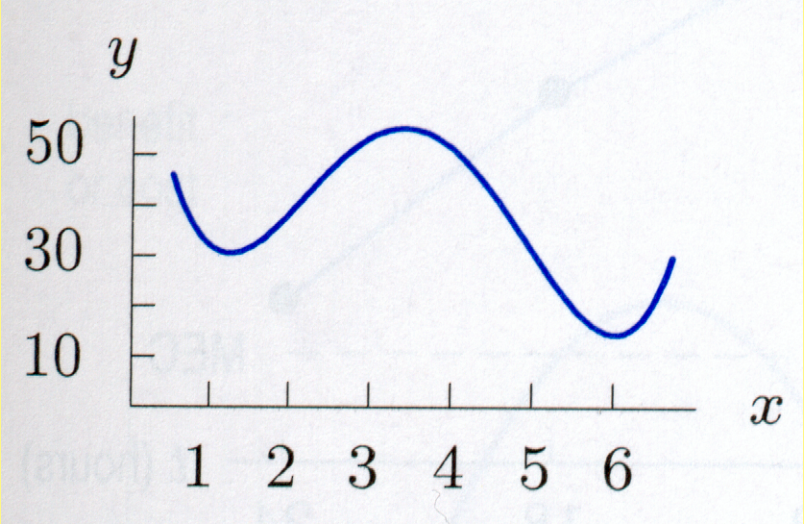

Where are the global maxima?

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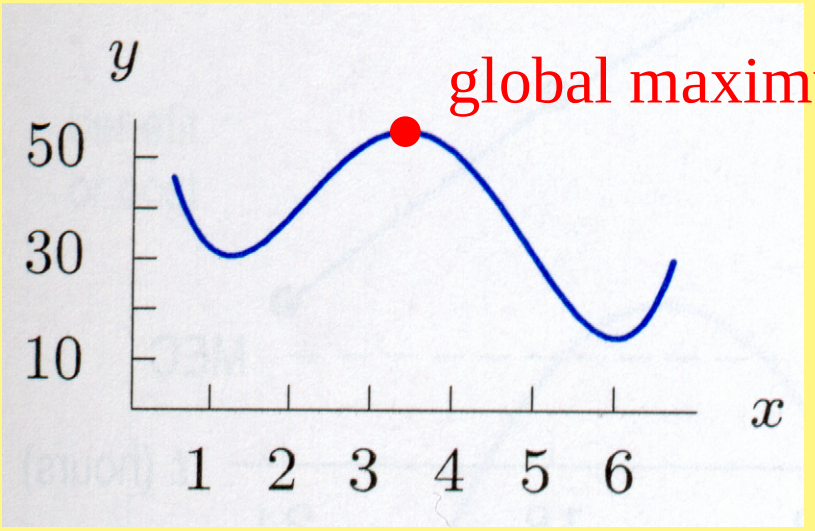

global maximum

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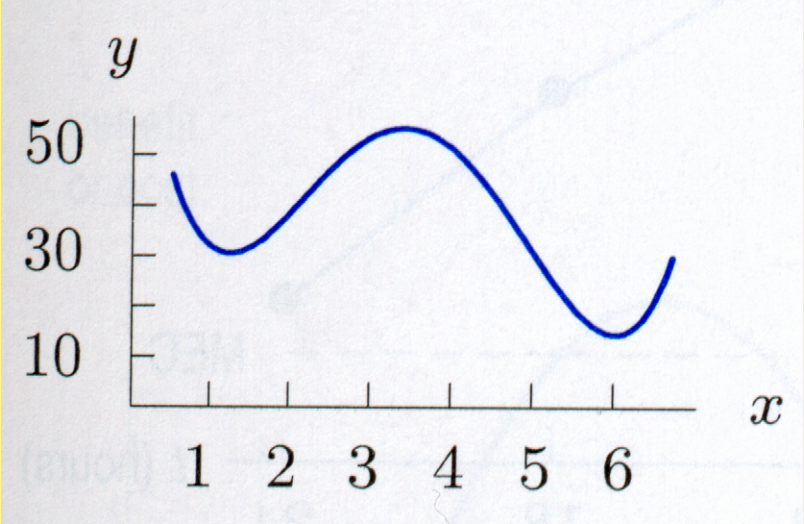

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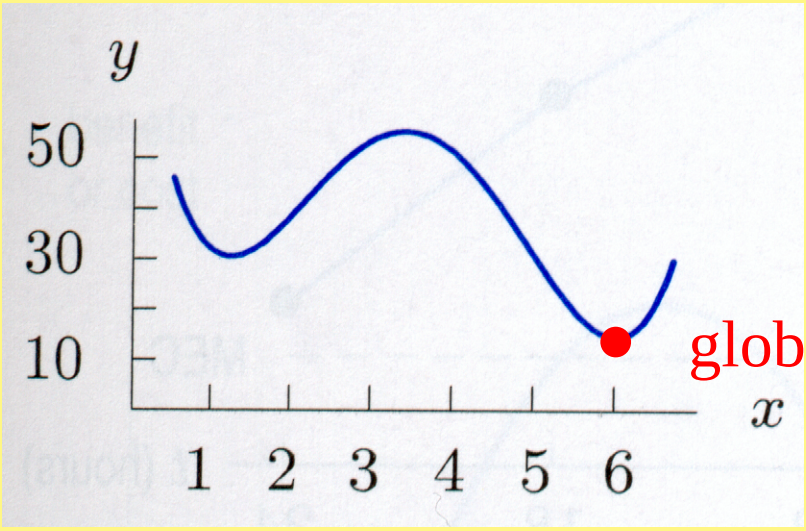
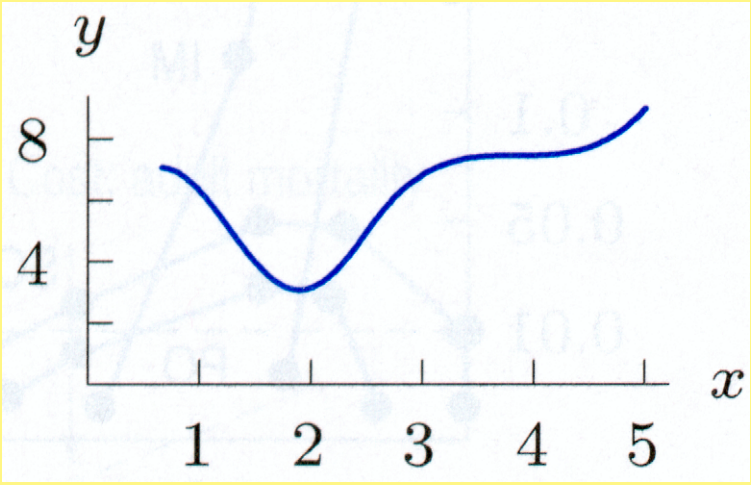


global minimum

Where are the global minima?

Problem 2:



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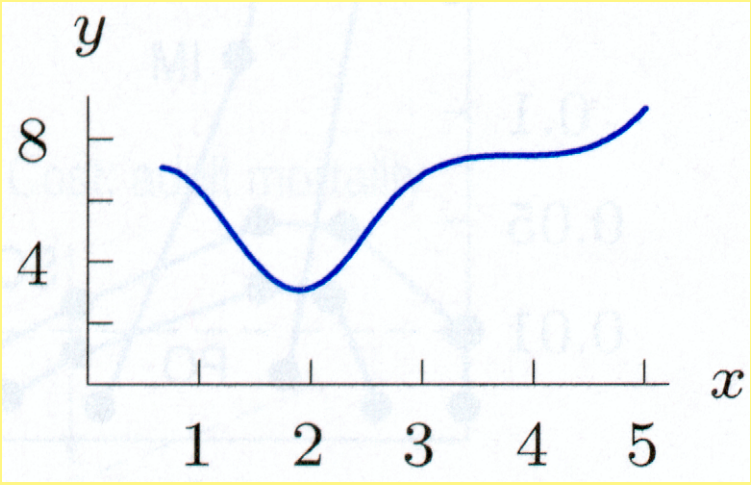

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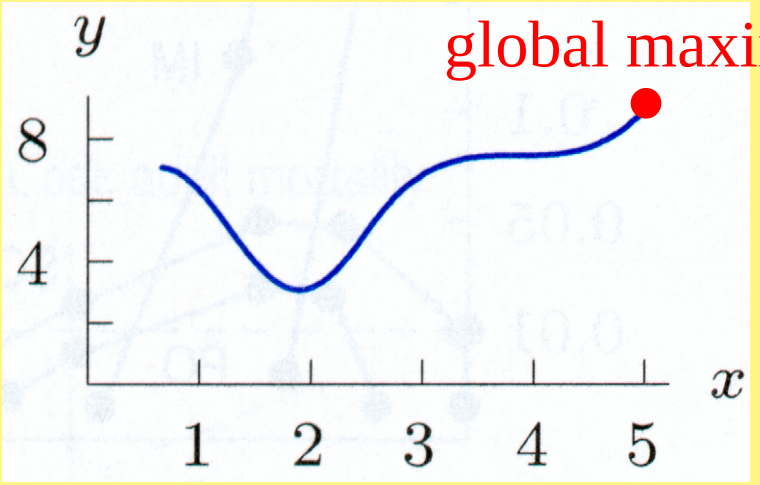

global maximum

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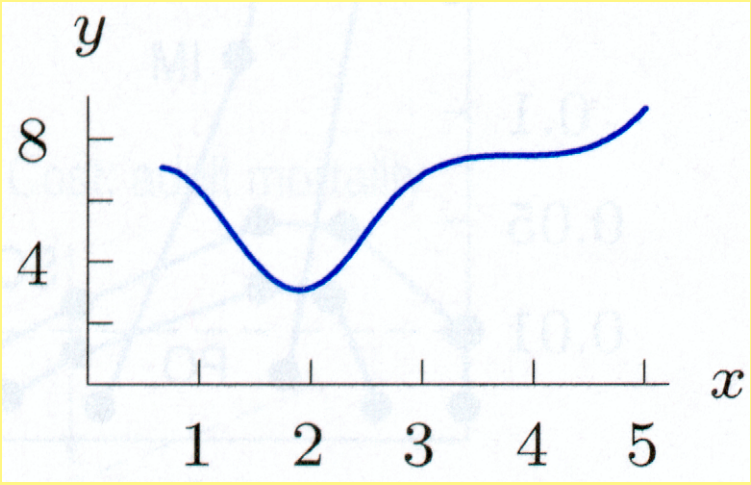

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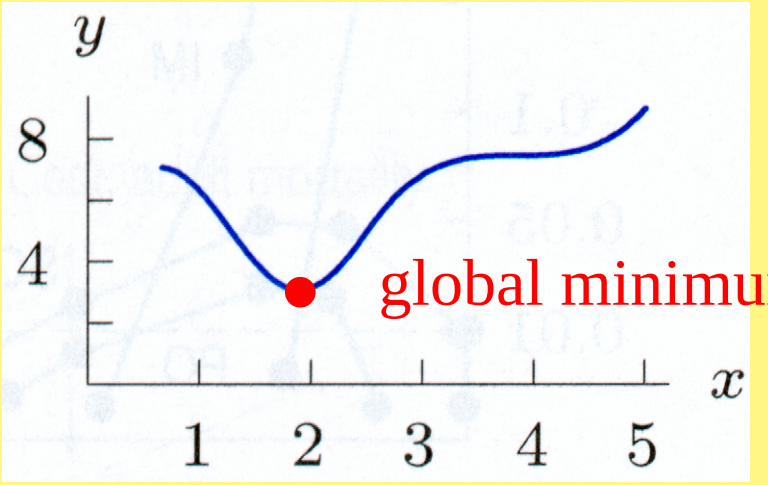

Where are the global minima?

Problem 7:

$$f(x) = 2x^3 - 9x^2 + 12x + 1 \qquad (-0.5 \leq x \leq 3)$$

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$$\begin{aligned} f'(x) &= 6x^2 - 18x + 12 \\ &= 6(x^2 - 3x + 2) \end{aligned}$$

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$$f(x) = 2x^3 - 9x^2 + 12x + 1 \qquad (-0.5 \leq x \leq 3)$$

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Points to Check: $-0.5,\ 1,\ 2,\ 3$

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Points to Check: $-0.5,\ 1,\ 2,\ 3$

Values of $f(x)$: $-7.5,\ 6,\ 5,\ 10$

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Global Maximum At:

Global Maximum Value:

Points to Check: $-0.5,\ 1,\ 2,\ 3$

Values of $f(x)$: $-7.5,\ 6,\ 5,\ 10$

Problem 7:

$$f(x) = 2x^3 - 9x^2 + 12x + 1 \qquad (-0.5 \leq x \leq 3)$$

Global Maximum At: 3

Global Maximum Value:

Points to Check: $-0.5,\ 1,\ 2,\ 3$

Values of $f(x)$: $-7.5,\ 6,\ 5,\ 10$

Problem 7:

$$f(x) = 2x^3 - 9x^2 + 12x + 1 \qquad (-0.5 \leq x \leq 3)$$

Global Maximum At: 3

Global Maximum Value: 10

Points to Check: $-0.5,\ 1,\ 2,\ 3$

Values of $f(x)$: $-7.5,\ 6,\ 5,\ 10$

Problem 7:

$$f(x) = 2x^3 - 9x^2 + 12x + 1 \qquad (-0.5 \leq x \leq 3)$$

Global Minimum At:

Global Minimum Value:

Points to Check: $-0.5,\ 1,\ 2,\ 3$

Values of $f(x)$: $-7.5,\ 6,\ 5,\ 10$

Problem 7:

$$f(x) = 2x^3 - 9x^2 + 12x + 1 \qquad (-0.5 \leq x \leq 3)$$

Global Minimum At: -0.5

Global Minimum Value:

Points to Check: $-0.5,\ 1,\ 2,\ 3$

Values of $f(x)$: $-7.5,\ 6,\ 5,\ 10$

Problem 7:

$$f(x) = 2x^3 - 9x^2 + 12x + 1 \qquad (-0.5 \leq x \leq 3)$$

Global Minimum At: -0.5

Global Minimum Value: -7.5

Points to Check: $-0.5,\ 1,\ 2,\ 3$

Values of $f(x)$: $-7.5,\ 6,\ 5,\ 10$

Problem 8:

$$f(x) = x^3 - 3x^2 - 9x + 15 \qquad (-5 \leq x \leq 4)$$

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$$\begin{aligned} f'(x) &= 3x^2 - 6x - 9 \\ &= 3(x^2 - 2x - 3) \end{aligned}$$

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$$f(x) = x^3 - 3x^2 - 9x + 15 \qquad (-5 \leq x \leq 4)$$

$$\begin{aligned} f'(x) &= 3x^2 - 6x - 9 \\ &= 3(x+1)(x-3) \end{aligned}$$

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Points to Check: $-5,\ -1,\ 3,\ 4$

Values of $f(x)$: $-140,\ 20,\ -12,\ -5$

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$$f(x) = x^3 - 3x^2 - 9x + 15 \qquad (-5 \leq x \leq 4)$$

Global Maximum At:

Global Maximum Value:

Points to Check: $-5,\ -1,\ 3,\ 4$

Values of $f(x)$: $-140,\ 20,\ -12,\ -5$

Problem 8:

$$f(x) = x^3 - 3x^2 - 9x + 15 \qquad (-5 \leq x \leq 4)$$

Global Maximum At: -1

Global Maximum Value: 20

Points to Check: $-5,\ -1,\ 3,\ 4$

Values of $f(x)$: $-140,\ 20,\ -12,\ -5$

Problem 8:

$$f(x) = x^3 - 3x^2 - 9x + 15 \qquad (-5 \leq x \leq 4)$$

Global Minimum At:

Global Minimum Value:

Points to Check: $-5,\ -1,\ 3,\ 4$

Values of $f(x)$: $-140,\ 20,\ -12,\ -5$

Problem 8:

$$f(x) = x^3 - 3x^2 - 9x + 15 \qquad (-5 \leq x \leq 4)$$

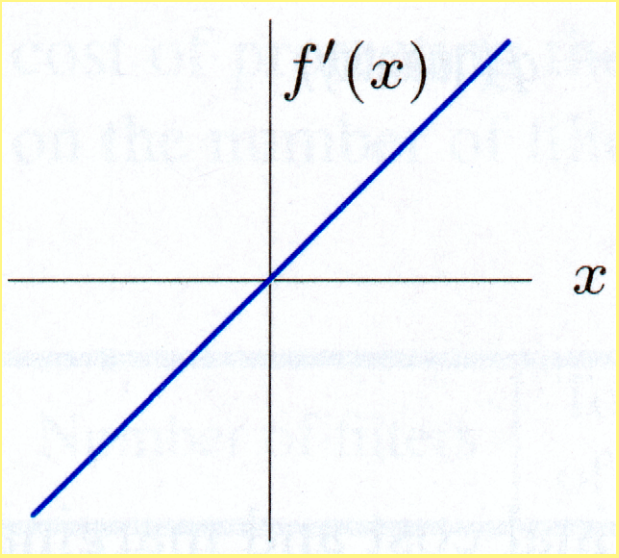
Global Minimum At: -5

Global Minimum Value: -140

Points to Check: $-5,\ -1,\ 3,\ 4$

Values of $f(x)$: $-140,\ 20,\ -12,\ -5$

Problem 9:



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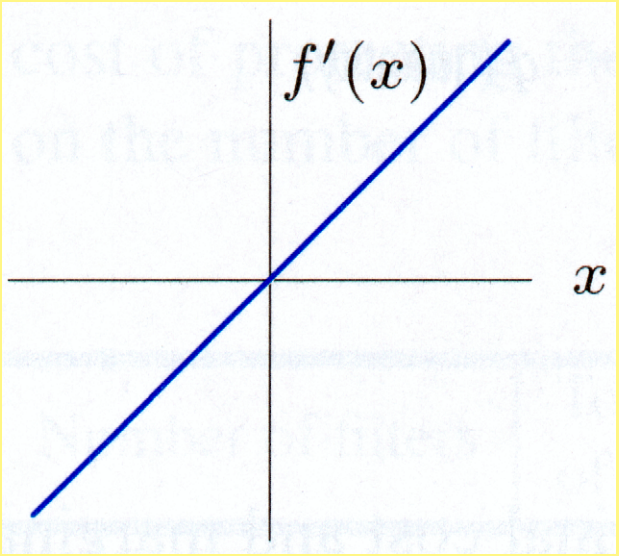

Where are the global maxima?

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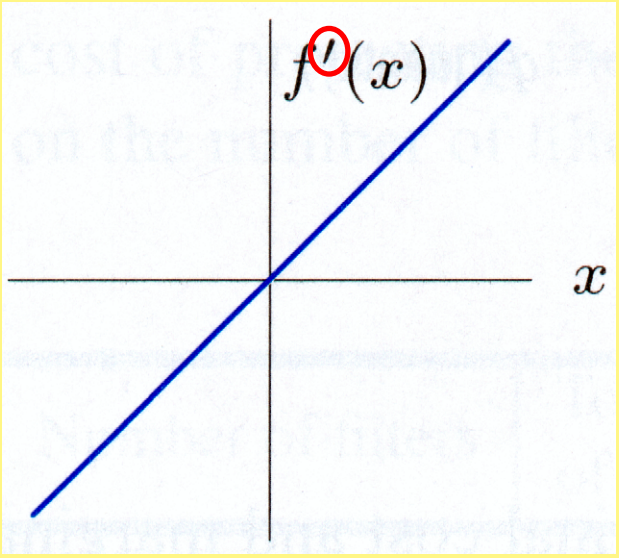

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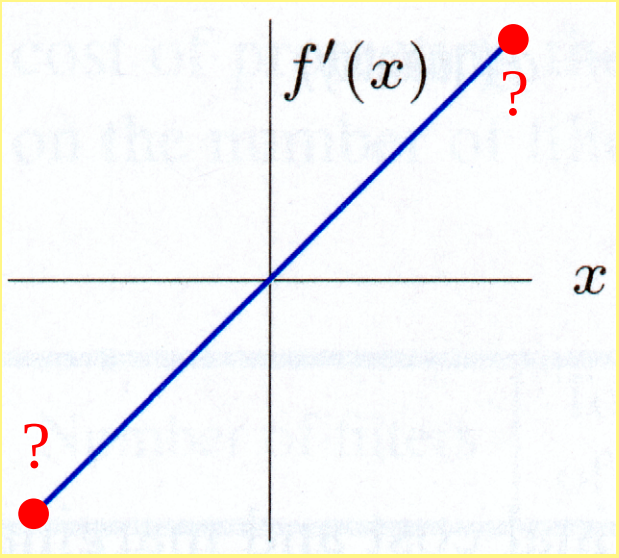

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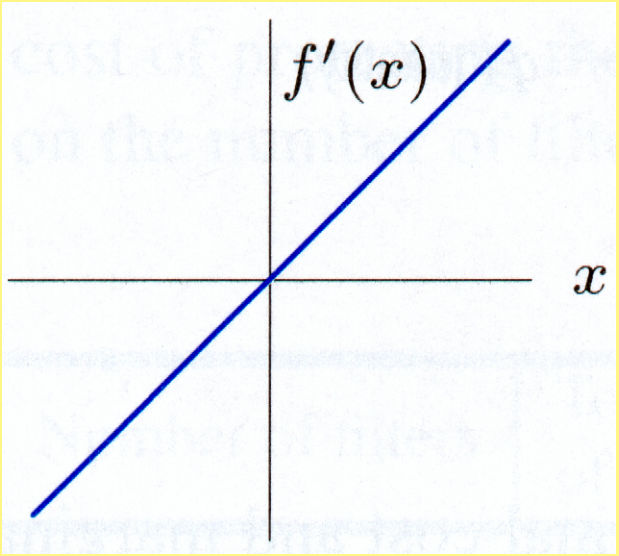

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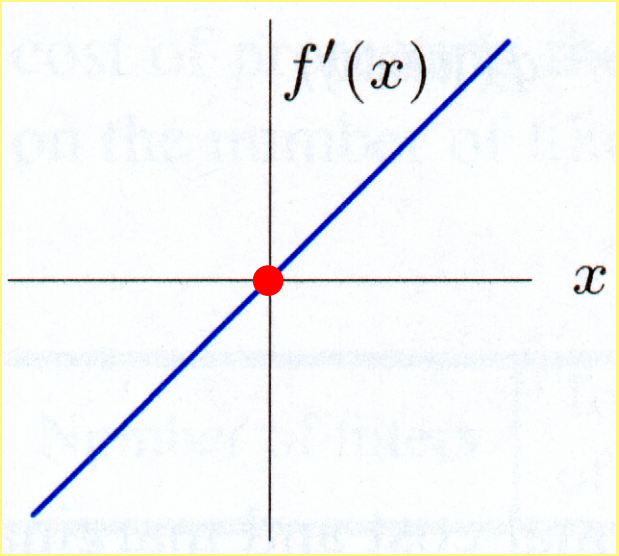

Where are the global minima?

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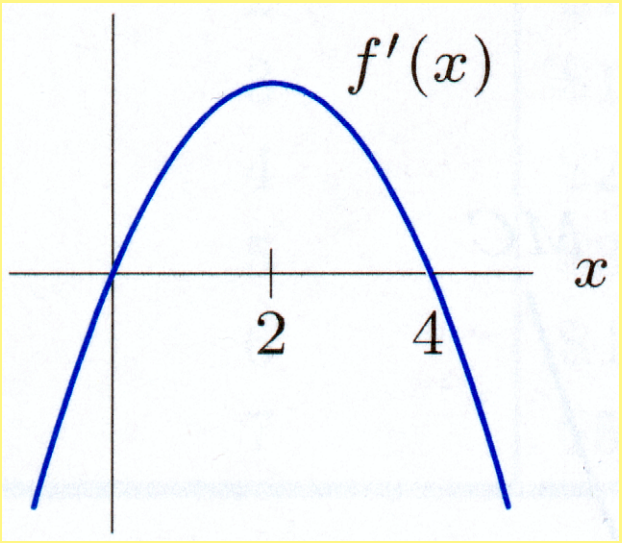

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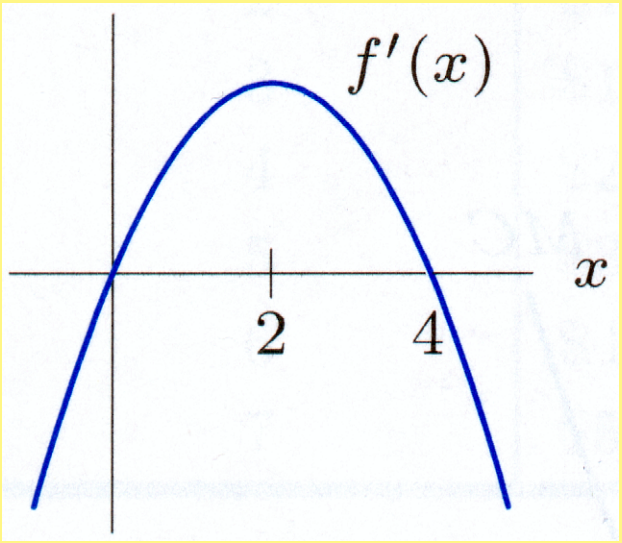

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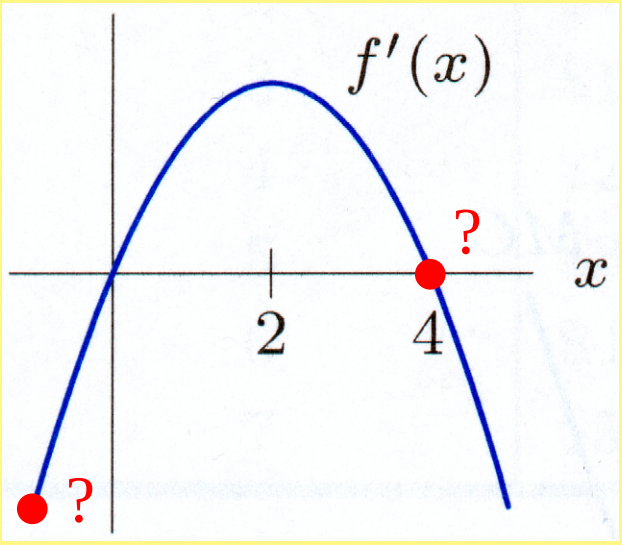

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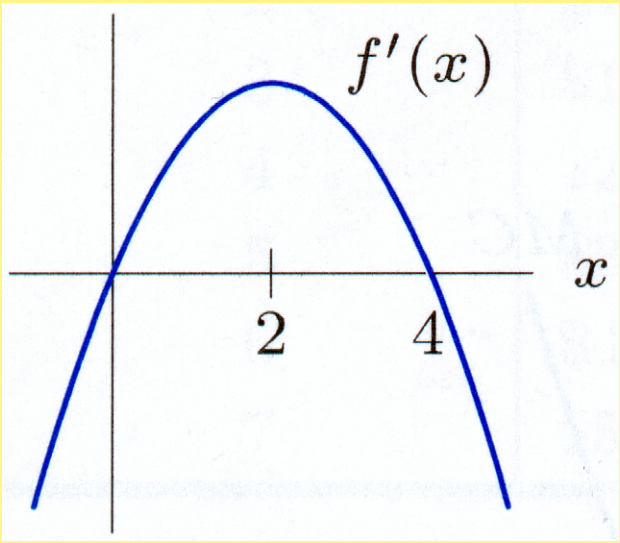

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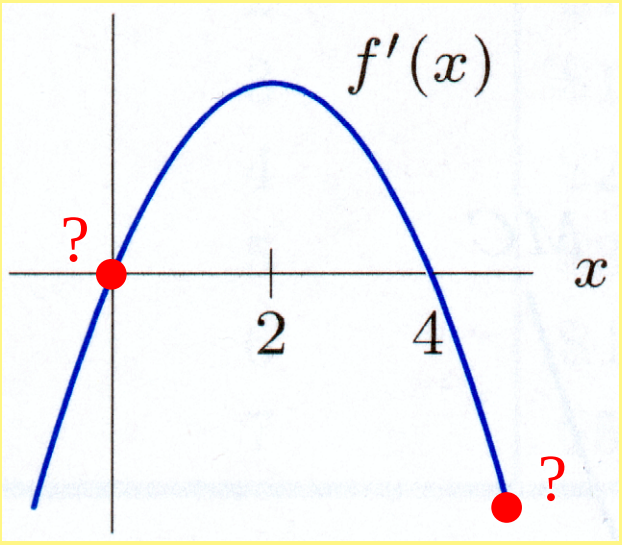
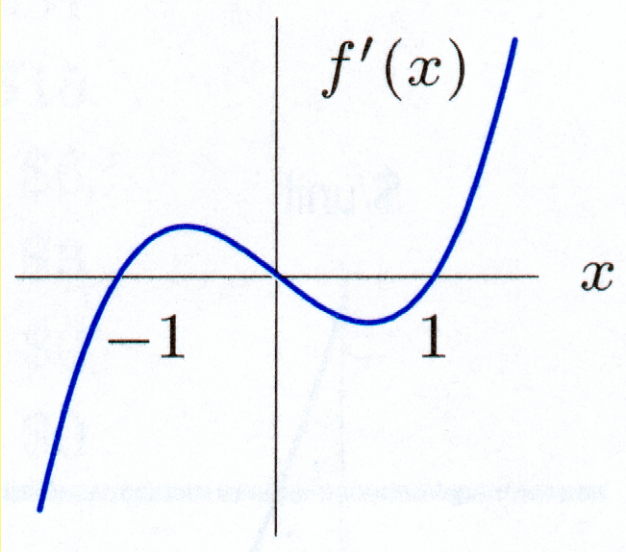
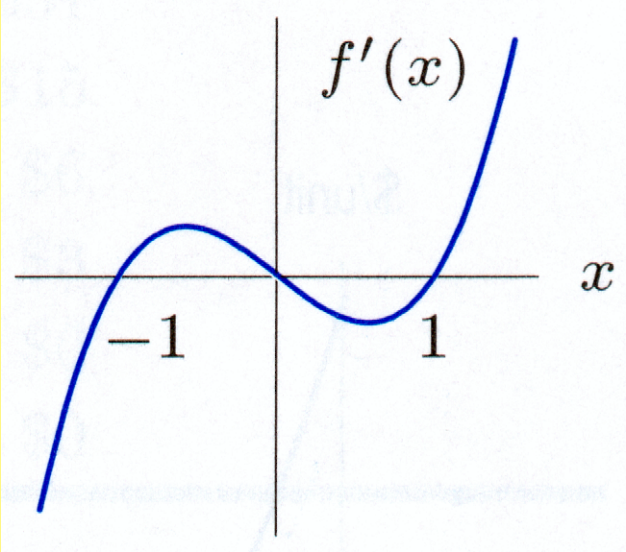


Where are the global minima?

Problem 12:



Problem 12:



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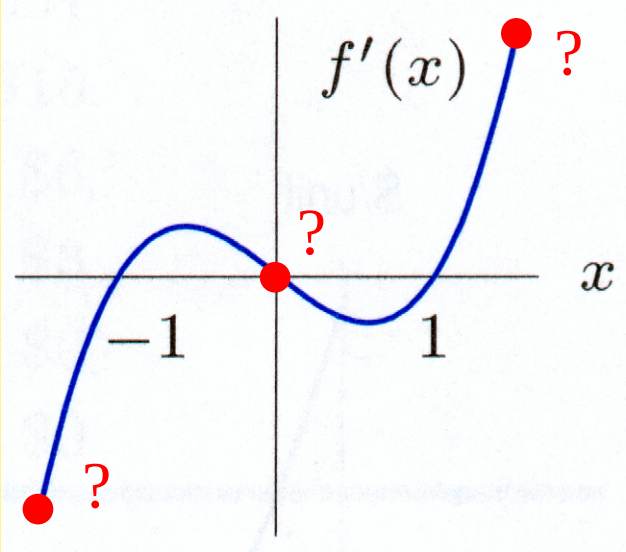

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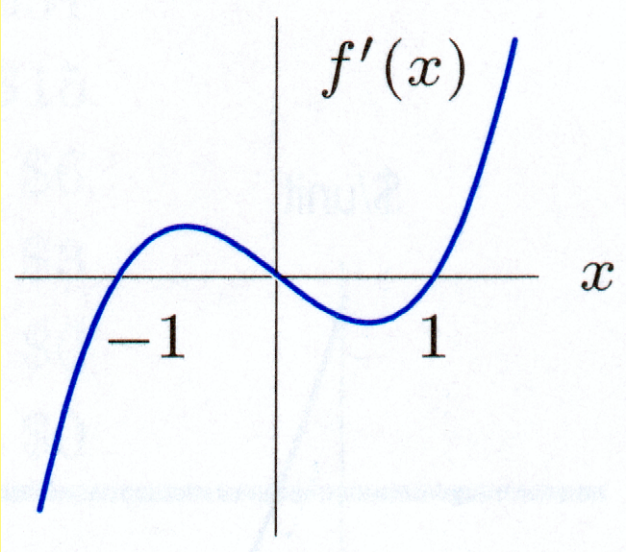

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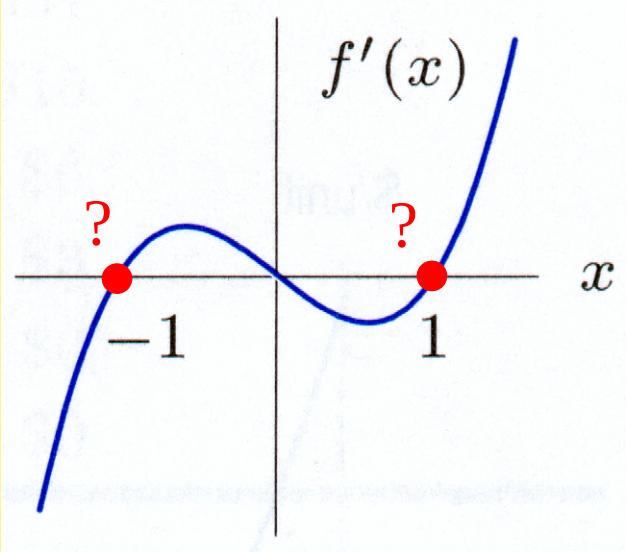

Where are the global minima?

Problem 13:

q	1000	2000	3000	4000	5000	6000
MR	78	76	74	72	70	68
MC	100	80	70	65	75	90

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What production levels could maximize profit?

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-22 -4 4 7 -5 -22

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Profit is maximized for $q = 1000$ or q in $[4000, 5000]$.

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MR	78	76	74	72	70	68
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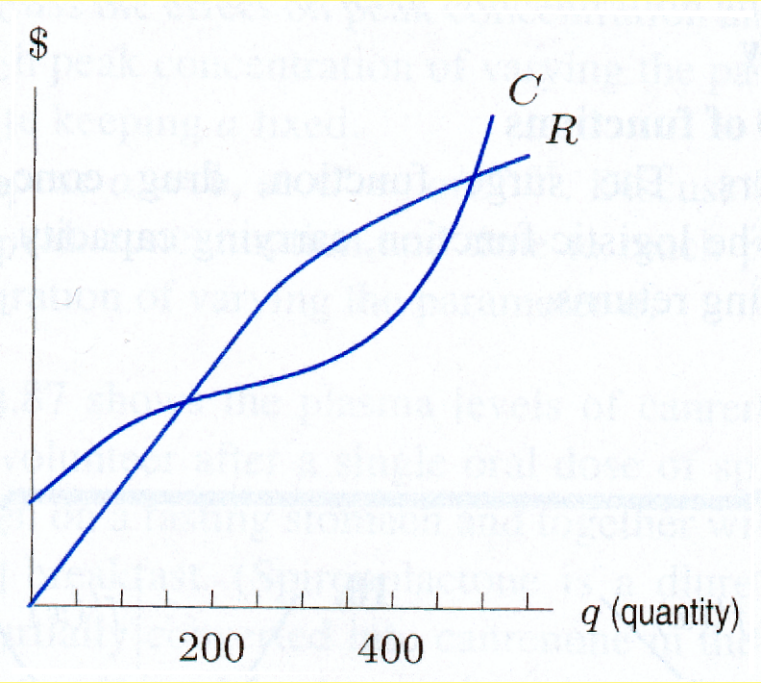
$-22 \quad -4 \quad 4 \quad 7 \quad -5 \quad -22$

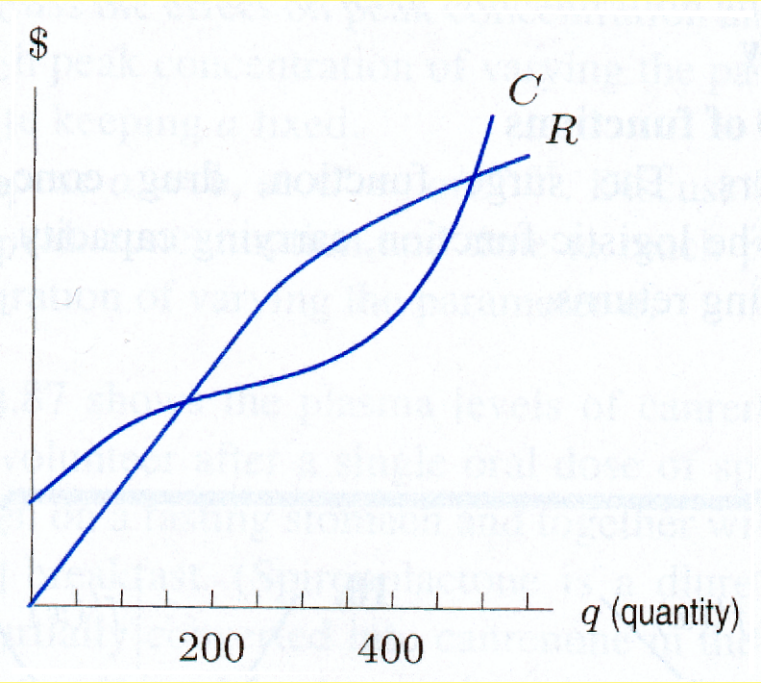
Profit is maximized for $q = 1000$ or q in $[4000, 5000]$.

Profit decreases, then increases, and then decreases.

Problem 17 (a):

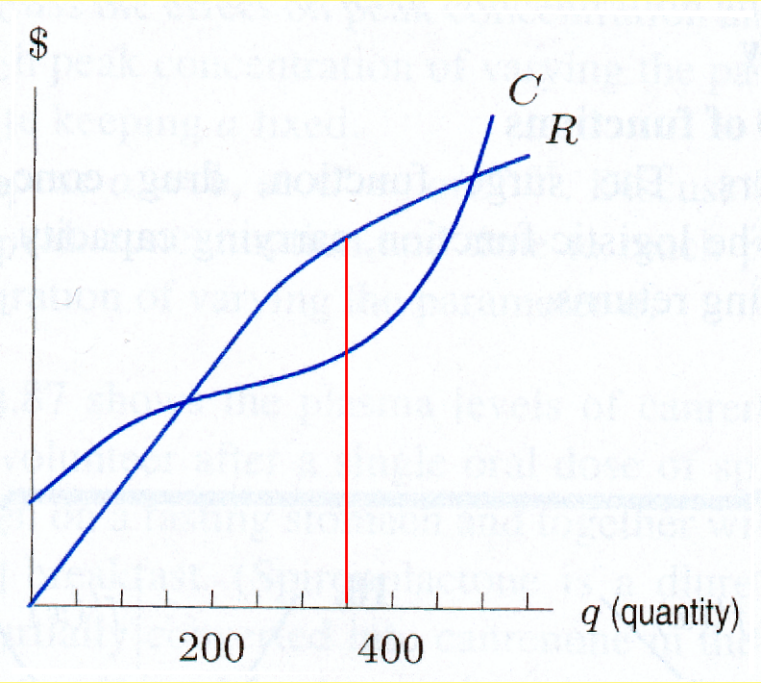


Problem 17 (a):



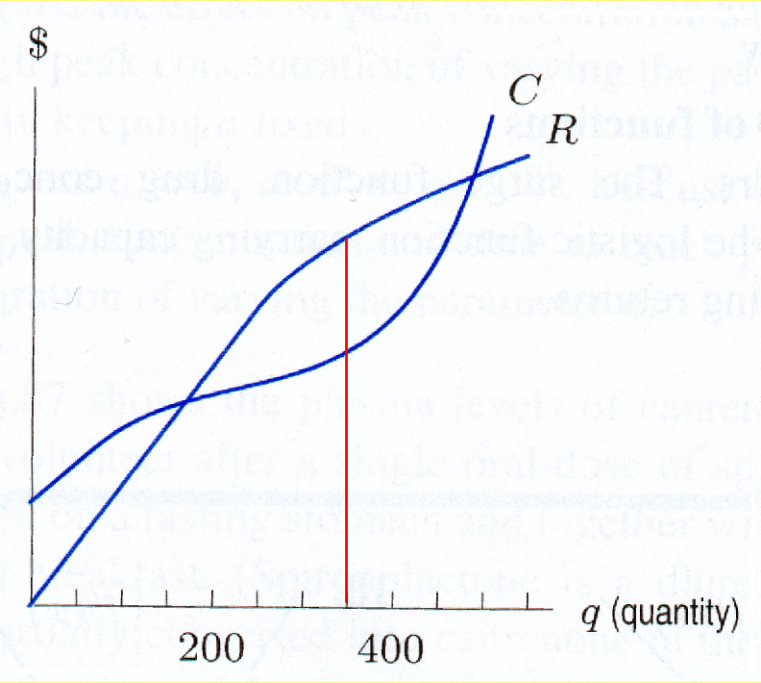
What production level maximizes profit?

Problem 17 (a):



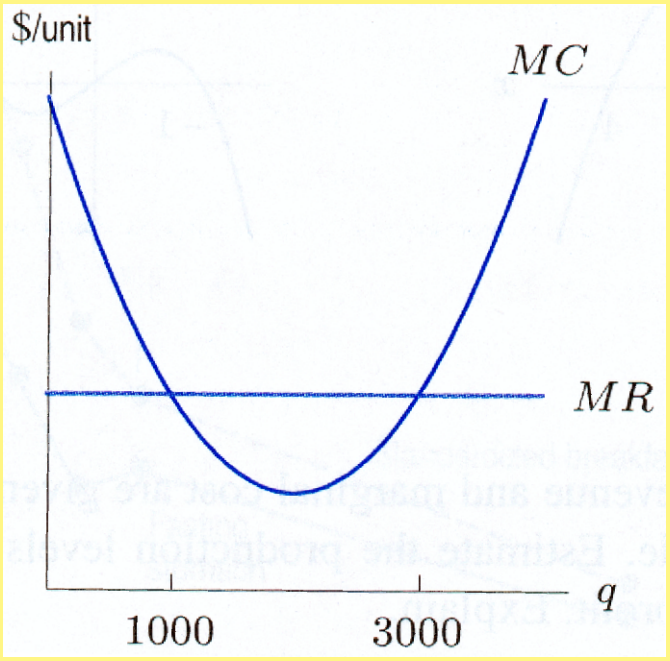
What production level maximizes profit?

Problem 17 (a):

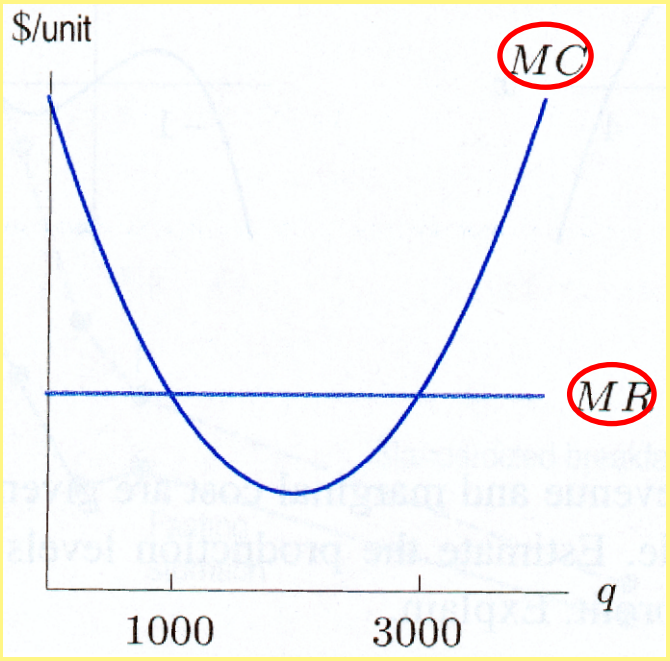


What production level maximizes profit? **350**

Problem 18:



Problem 18:



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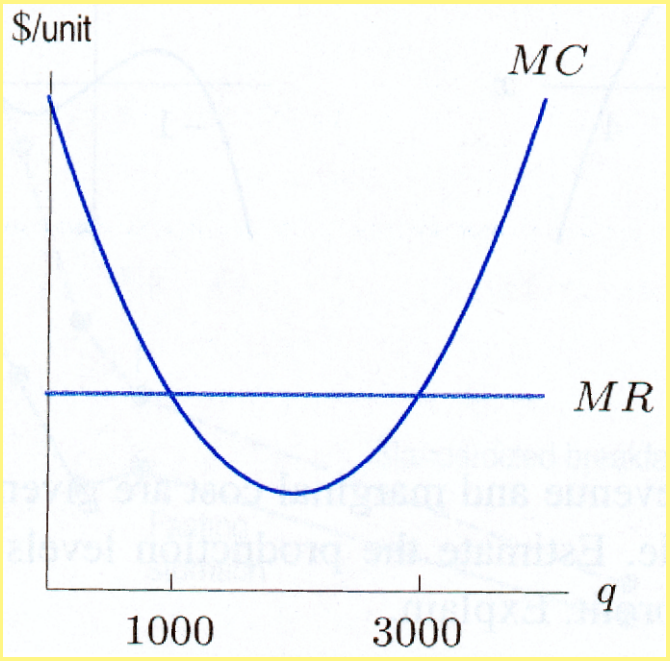
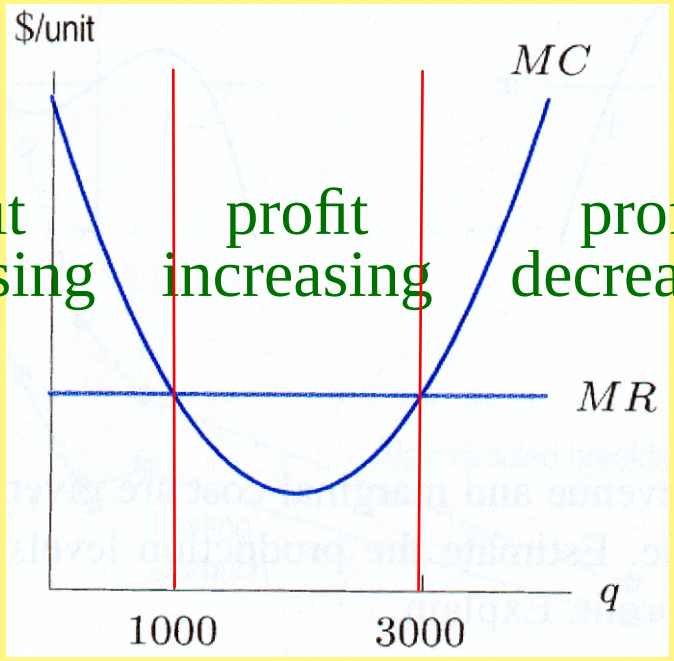


What production level could maximize profit?

Problem 18:



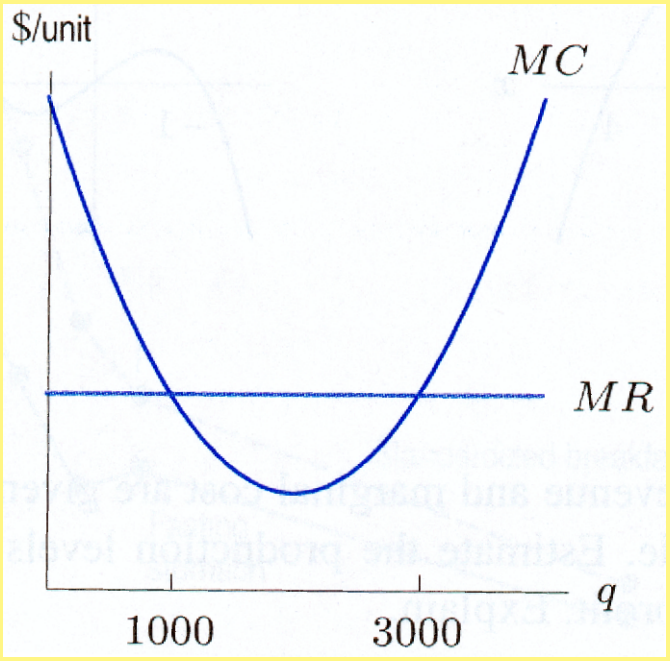
profit decreasing

profit increasing

profit decreasing

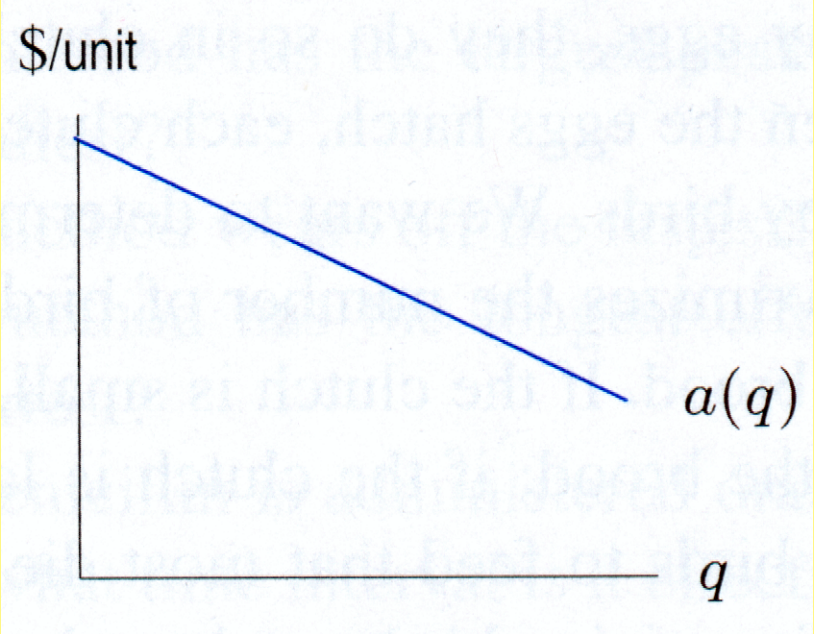
What production level could maximize profit?

Problem 18:

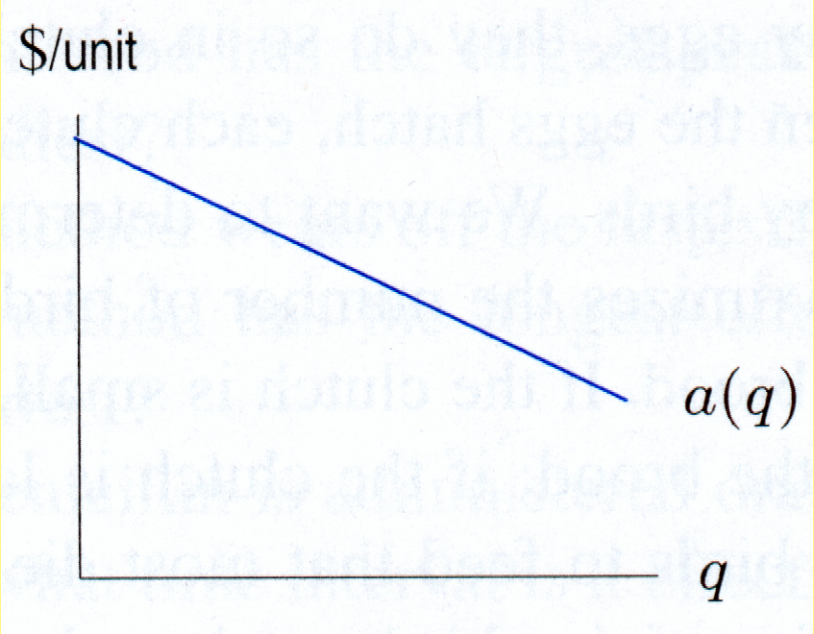


What production level could maximize profit? **0** or **3000**

Problem 30:



Problem 30:



$a(q) = b + mq$ $\qquad\qquad$ $C'(q) = ?$

Problem 30:

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$$\frac{C(q)}{q} = b + mq \qquad\qquad C'(q) = ?$$

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$$C'(q) = b \cdot 1 + m \cdot 2q$$

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$$\frac{C(q)}{q} = b + mq \qquad\qquad C'(q) = ?$$

$$C(q) = bq + mq^2$$

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The graph of the marginal cost is a straight line

Problem 30:

$$\frac{C(q)}{q} = b + mq \qquad\qquad C'(q) = ?$$

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The graph of the marginal cost is a straight line with vertical intercept b

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$$\frac{C(q)}{q} = b + mq \qquad\qquad C'(q) = ?$$

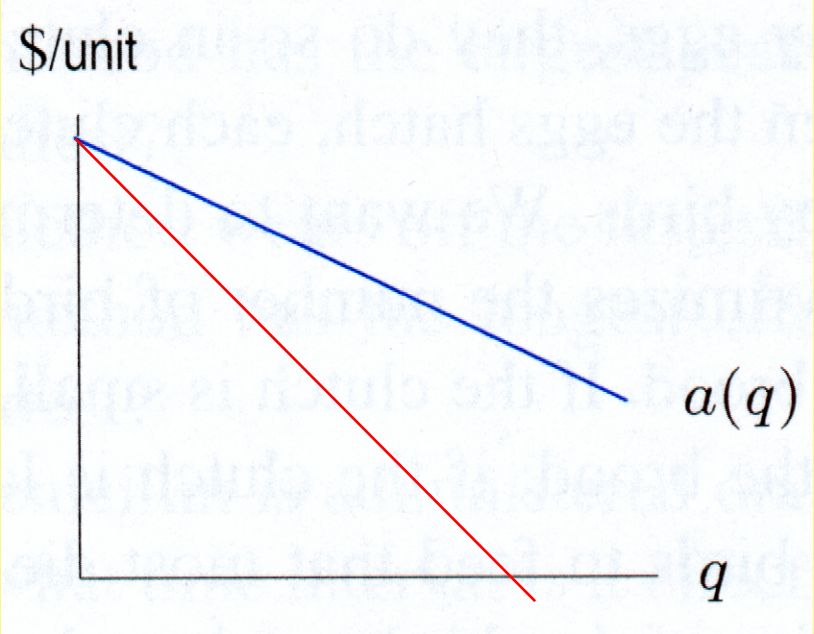
$$C(q) = bq + mq^2$$

$$C'(q) = b \cdot 1 + m \cdot 2q$$

$$C'(q) = b + 2qm$$

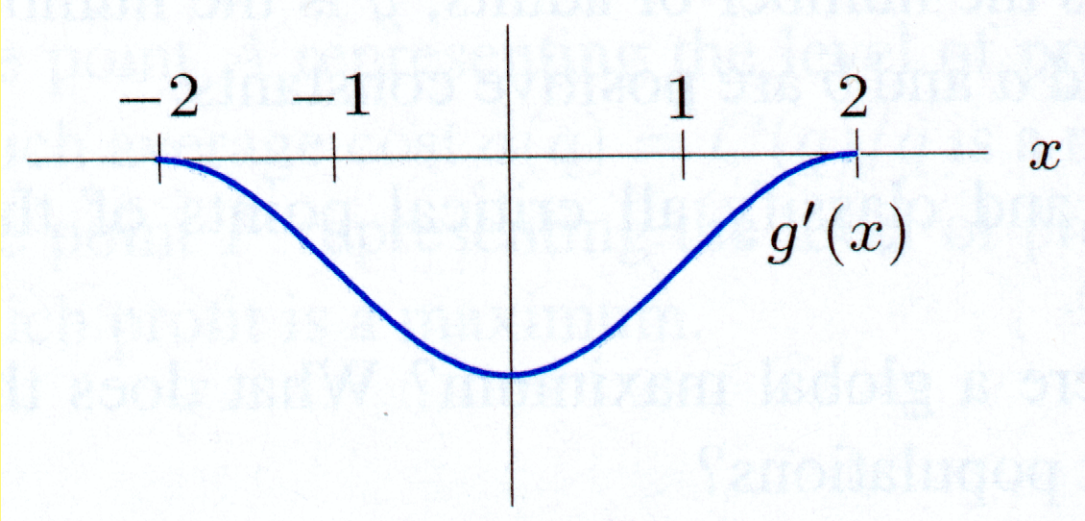
The graph of the marginal cost is a straight line with vertical intercept b and negative slope $2m$.

Problem 30:

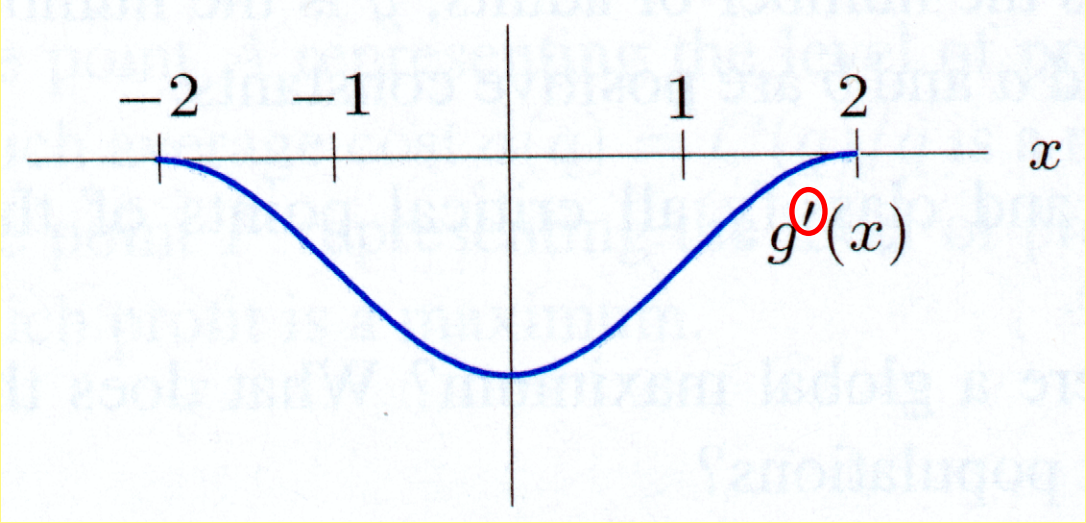


Marginal Cost in Red

Problem 32 (c):

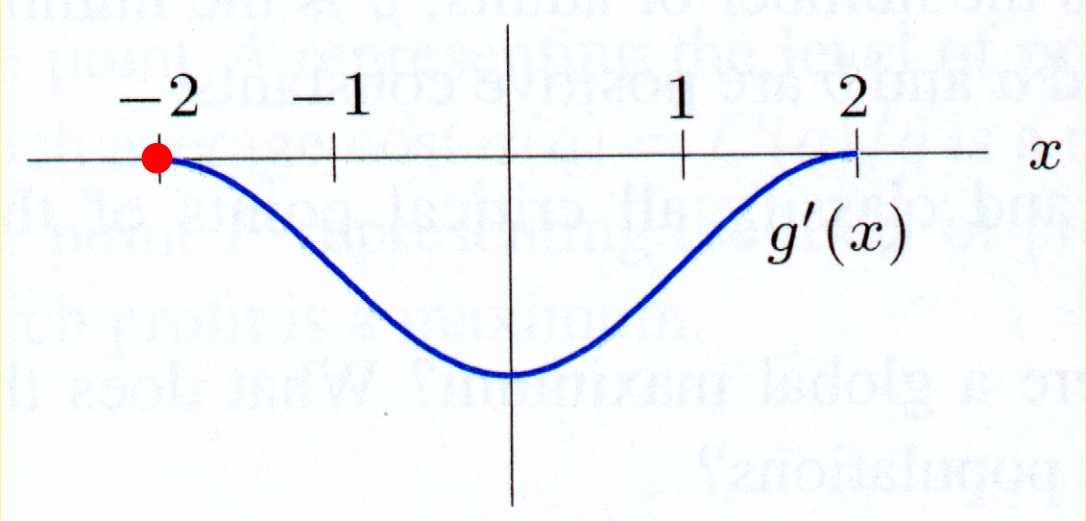


Problem 32 (c):



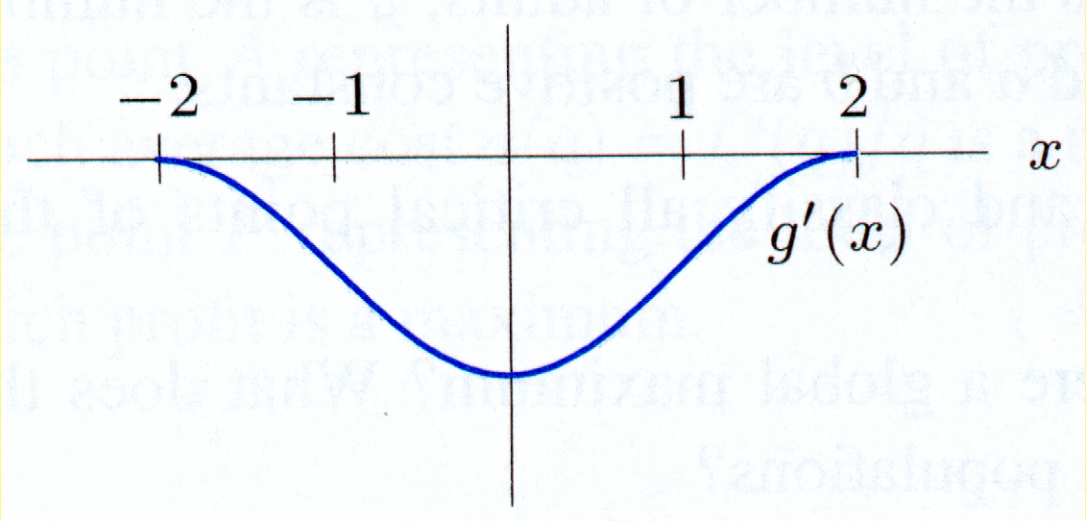
Where are the global maxima?

Problem 32 (c):



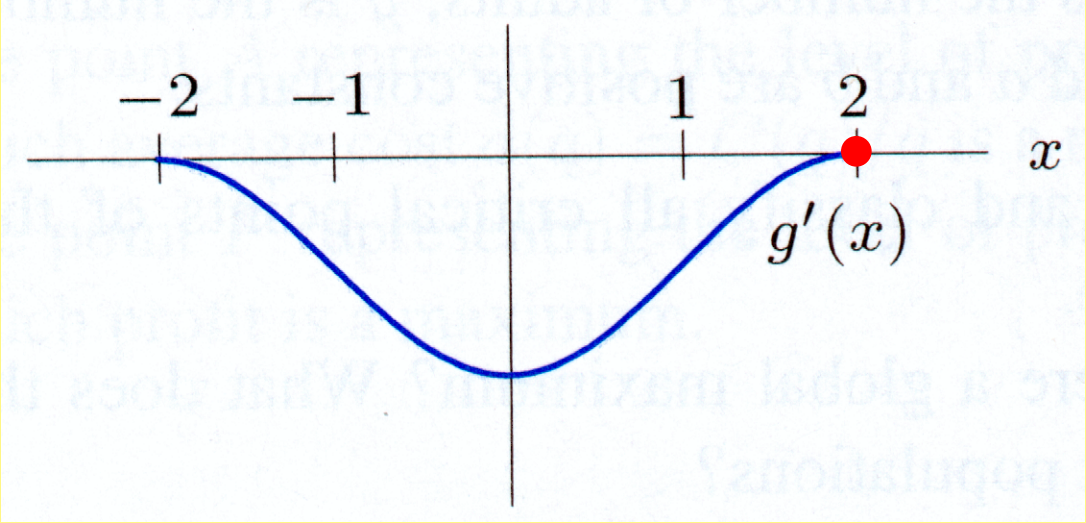
Where are the global maxima?

Problem 32 (c):



Where are the global minima?

Problem 32 (c):



Where are the global minima?

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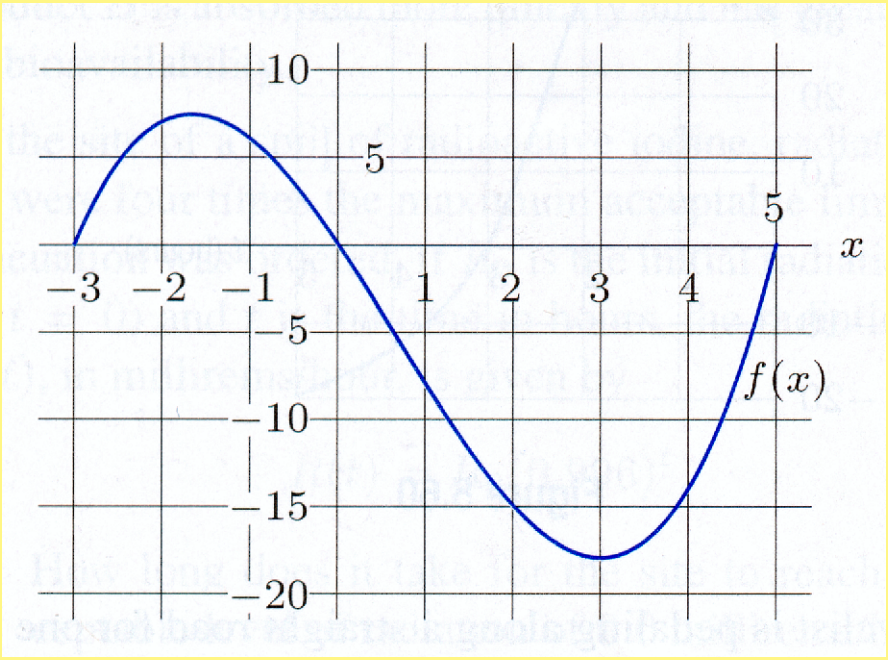
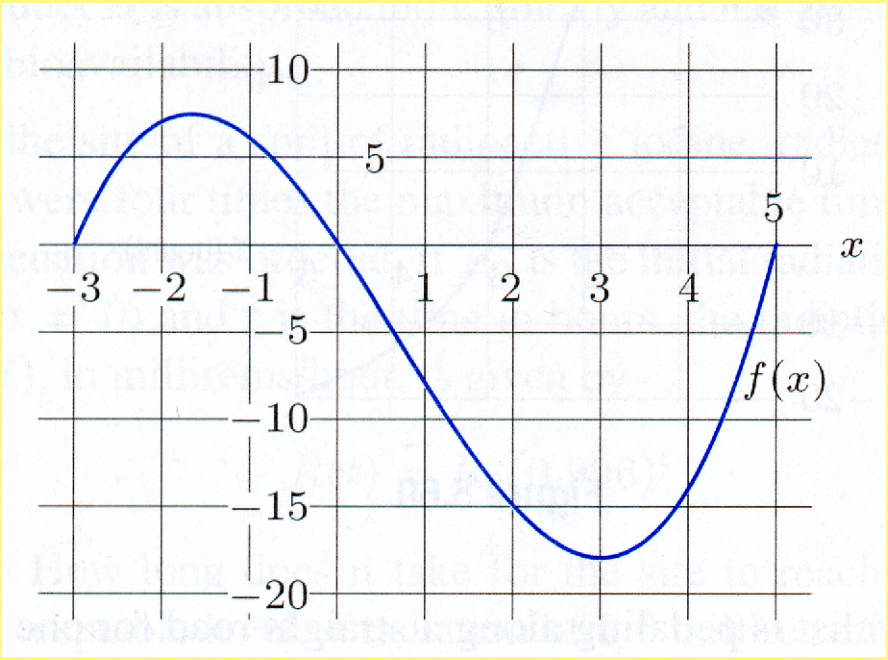
Problem 3:



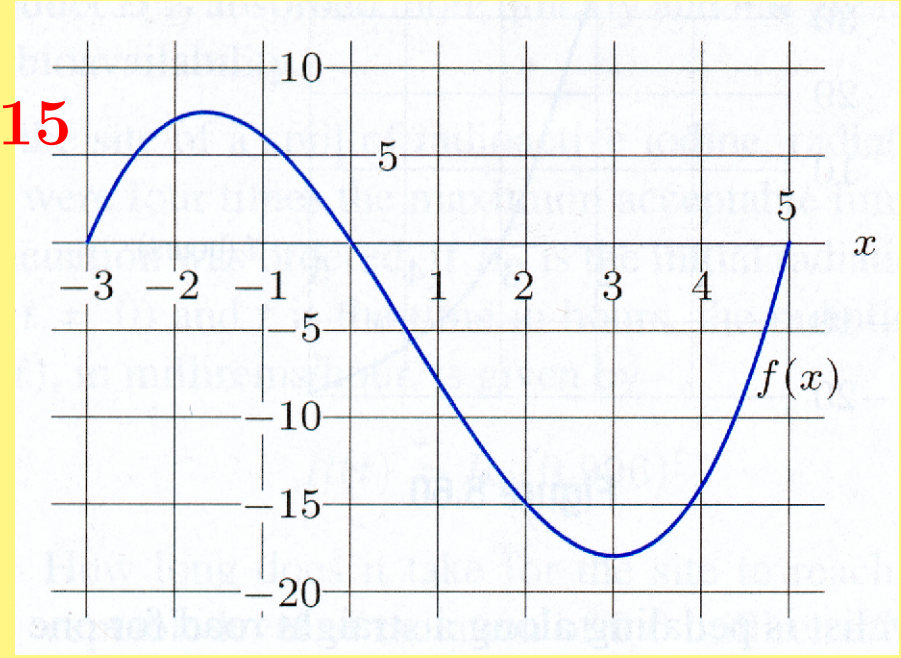
Problem 3:



Estimate $\int_{-3}^{5} f(x)\,dx$?

Problem 3:

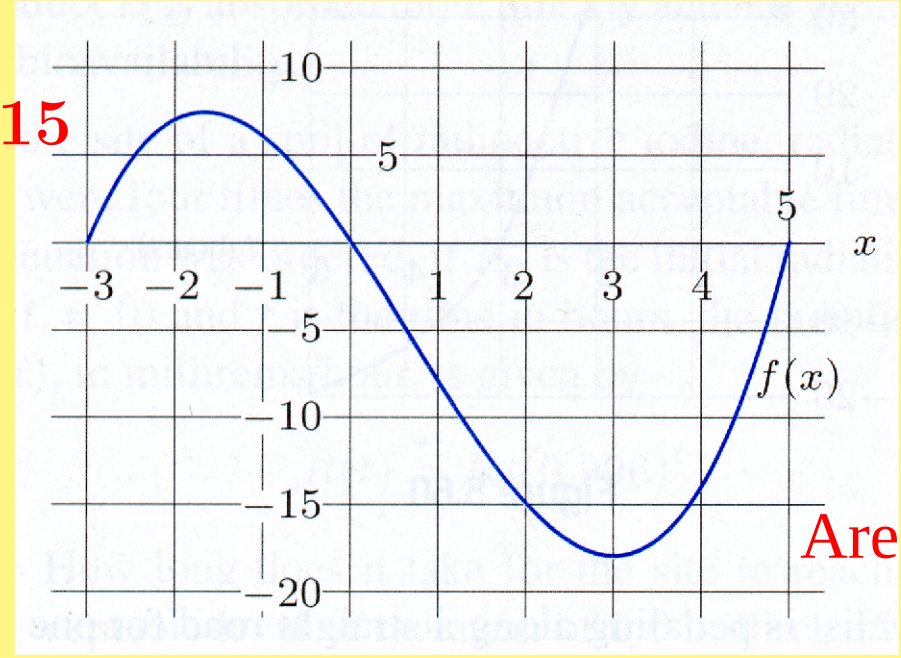
Area $= 15$



Estimate $\int_{-3}^{5} f(x)dx$?

Problem 3:

Area $= 15$

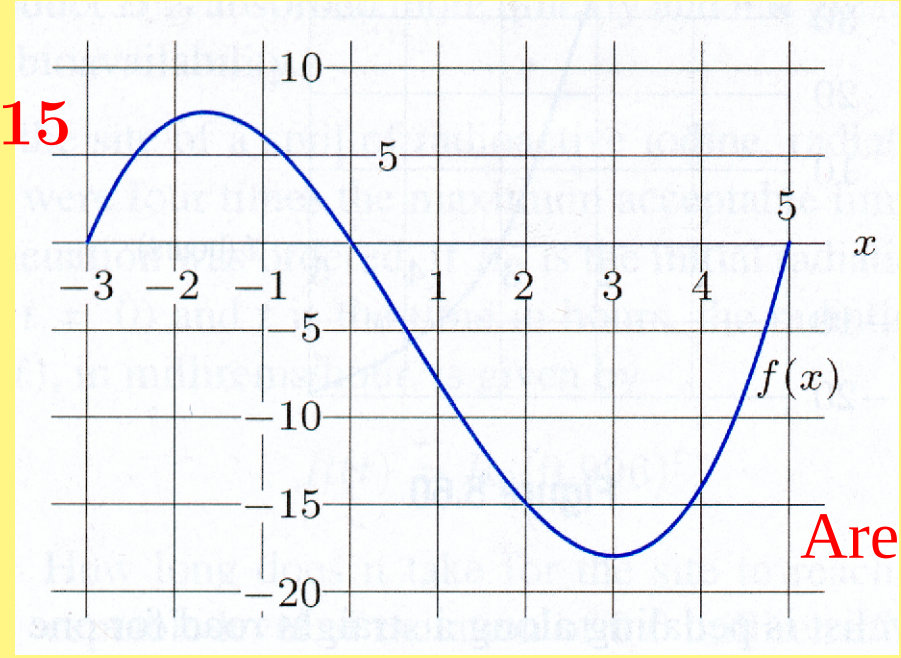


Area $= 55$

Estimate $\int_{-3}^{5} f(x)dx$?

Problem 3:

Area $= 15$

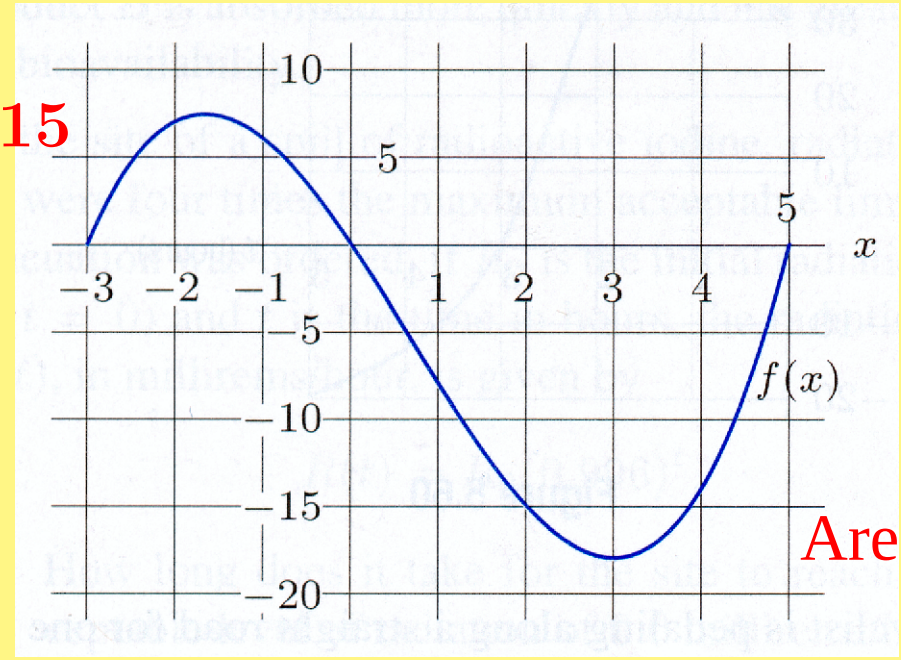


Area $= 55$

Estimate $\int_{-3}^{5} f(x)dx$? $\quad 15 - 55$

Problem 3:

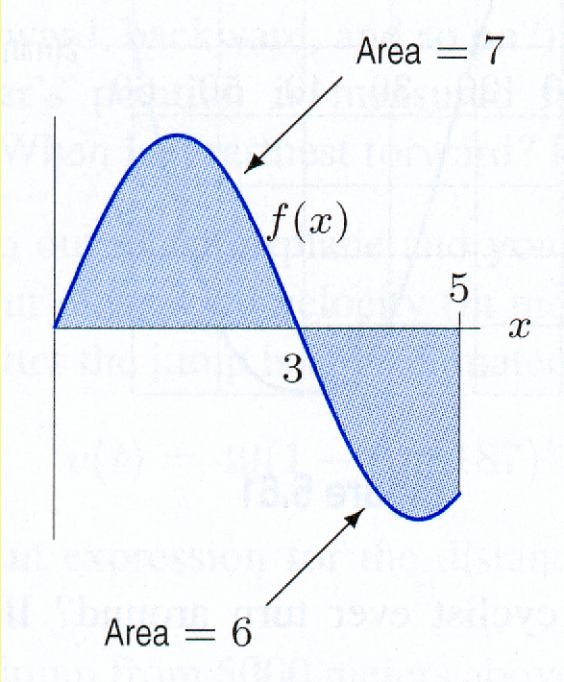
Area $= 15$



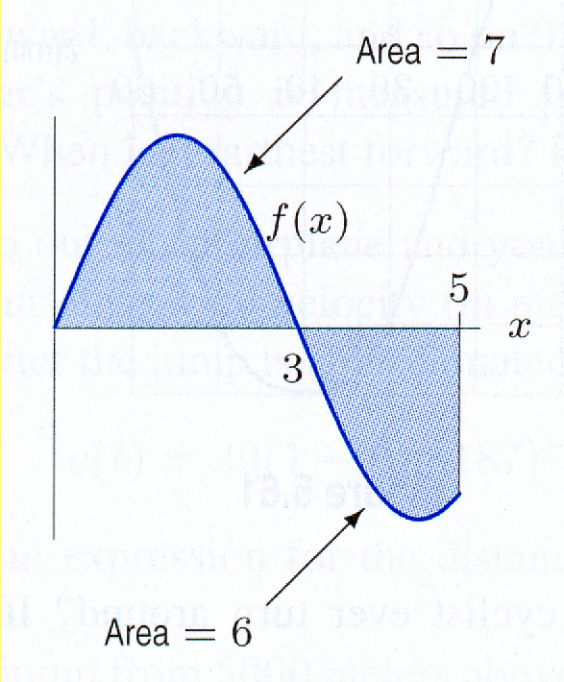
Area $= 55$

Estimate $\int_{-3}^{5} f(x)dx$? $\quad 15 - 55 = -40$

Problem 4:

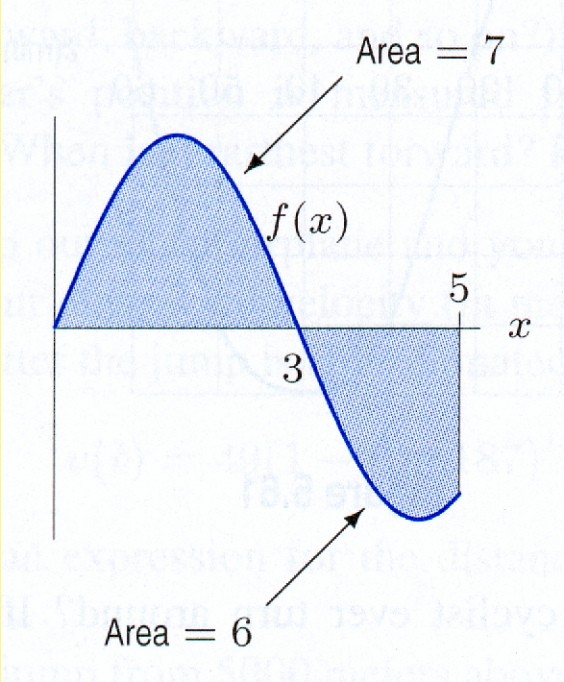


Problem 4:



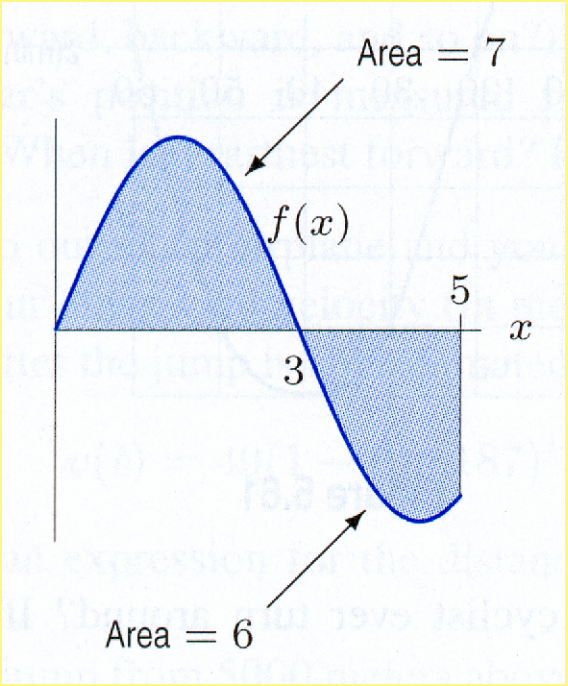
What's the area of the shaded region?

Problem 4:



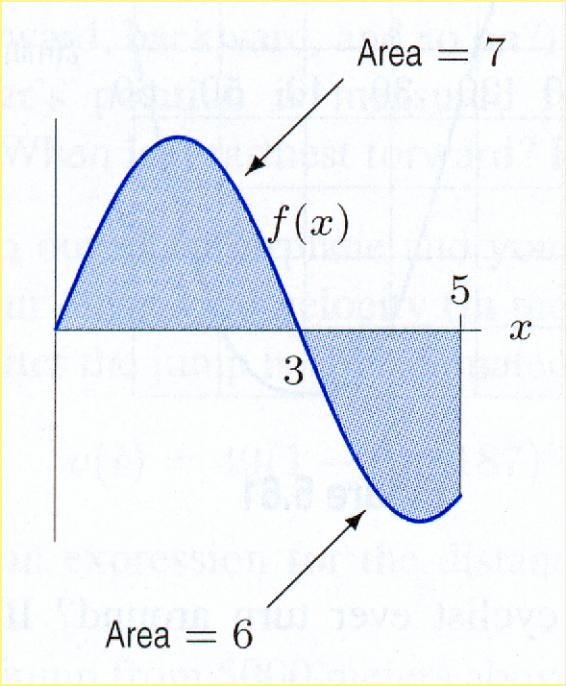
What's the area of the shaded region? $\mathbf{7 + 6 = 13}$

Problem 4:



What's $\int_0^5 f(x)dx$?

Problem 4:



What's $\int_0^5 f(x)dx$? $7 - 6 = 1$

Problem 6:

$$\int_{1}^{5} (x^2 + 1) dx = ?$$

Problem 6:

$$\int_{1}^{5} (x^2 + 1) dx = ?$$

Problem 6:

$$\int_1^5 (x^2 + 1)dx = ?$$

$$F(x) = x^3$$

Problem 6:

$$\int_1^5 (x^2+1)dx = ?$$

$$F(x) = \frac{1}{3} \cdot x^3$$

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Problem 6:

$$\int_1^5 (x^2+1)dx = ?$$

$$F(x) = \frac{1}{3} \cdot x^3 + x$$

Problem 6:

$$\int_1^5 (x^2 + 1)dx = ?$$

$$F(x) = \frac{1}{3} \cdot x^3 + x$$

$$F(5) - F(1)$$

Problem 6:

$$\int_1^5 (x^2+1)dx = ?$$

$$F(x) = \frac{1}{3}\cdot x^3 + x$$

$$F(5) - F(1) = \left(\frac{1}{3}\cdot 5^3 + 5\right) - \left(\frac{1}{3}\cdot 1^3 + 1\right)$$

Problem 6:

$$\int_1^5 (x^2+1)dx = ?$$

$$F(x) = \frac{1}{3}\cdot x^3 + x$$

$$F(5) - F(1) = \left(\frac{1}{3}\cdot 5^3 + 5\right) - \left(\frac{1}{3}\cdot 1^3 + 1\right)$$

$$= \frac{125}{3} + 5 - \frac{1}{3} - 1$$

Problem 6:

$$\int_1^5 (x^2+1)dx = ?$$

$$F(x) = \frac{1}{3}\cdot x^3 + x$$

$$F(5) - F(1) = \left(\frac{1}{3}\cdot 5^3 + 5\right) - \left(\frac{1}{3}\cdot 1^3 + 1\right)$$

$$= \frac{125}{3} + 5 - \frac{1}{3} - 1 = \frac{124}{3} + 4$$

Problem 6:

$$\int_1^5 (x^2+1)dx = ?$$

$$F(x) = \frac{1}{3}\cdot x^3 + x$$

$$F(5) - F(1) = \left(\frac{1}{3}\cdot 5^3 + 5\right) - \left(\frac{1}{3}\cdot 1^3 + 1\right)$$

$$= \frac{125}{3} + 5 - \frac{1}{3} - 1 = \frac{124}{3} + 4 = \frac{136}{3}$$

Problem 10:

$$\int_1^3 \frac{z^2+1}{z}\,dz = ?$$

Problem 10:

$$\int_1^3 \frac{z^2+1}{z}\,dz = ?$$

$$\int_1^3 z + \frac{1}{z}\,dz = ?$$

Problem 10:

$$\int_1^3 \frac{z^2+1}{z}\,dz = ?$$

$$\int_1^3 z+\frac{1}{z}\,dz = ?$$

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Problem 10:

$$\int_1^3 z + \frac{1}{z}\, dz = ?$$

Problem 10:

$$\int_1^3 z + \frac{1}{z}\, dz = ?$$

$$F(z) =$$

Problem 10:

$$\int_1^3 z + \frac{1}{z}\, dz = ?$$

$$F(z) = z^2$$

Problem 10:

$$\int_1^3 z + \frac{1}{z}\, dz = ?$$

$$F(z) = \frac{1}{2} \cdot z^2$$

Problem 10:

$$\int_1^3 z + \frac{1}{z}\, dz = ?$$

$$F(z) = \frac{1}{2} \cdot z^2 +$$

Problem 10:

$$\int_1^3 z + \frac{1}{z}\, dz = ?$$

$$F(z) = \frac{1}{2} \cdot z^2 + \ln z$$

Problem 10:

$$\int_1^3 z + \frac{1}{z}\, dz = ?$$

$$F(z) = \frac{1}{2} \cdot z^2 + \ln z$$

$F(3) - F(1)$

Problem 10:

$$\int_1^3 z + \frac{1}{z}\, dz = ?$$

$$F(z) = \frac{1}{2} \cdot z^2 + \ln z$$

$$F(3) - F(1) = \left(\frac{1}{2} \cdot 3^2 + \ln 3\right) - \left(\frac{1}{2} \cdot 1^2 + \ln 1\right)$$

Problem 10:

$$\int_1^3 z + \frac{1}{z}\, dz = ?$$

$$F(z) = \frac{1}{2} \cdot z^2 + \ln z$$

$$F(3) - F(1) = \left(\frac{1}{2} \cdot 3^2 + \ln 3\right) - \left(\frac{1}{2} \cdot 1^2 + \ln 1\right)$$

$$= \left(\frac{9}{2} + \ln 3\right) - \left(\frac{1}{2} + \ln 1\right)$$

Problem 10:

$$\int_1^3 z + \frac{1}{z}\, dz = ?$$

$$F(z) = \frac{1}{2} \cdot z^2 + \ln z$$

$$F(3) - F(1) = \left(\frac{1}{2} \cdot 3^2 + \ln 3\right) - \left(\frac{1}{2} \cdot 1^2 + \ln 1\right)$$

$$= \left(\frac{9}{2} + \ln 3\right) - \left(\frac{1}{2}\right)$$

Problem 10:

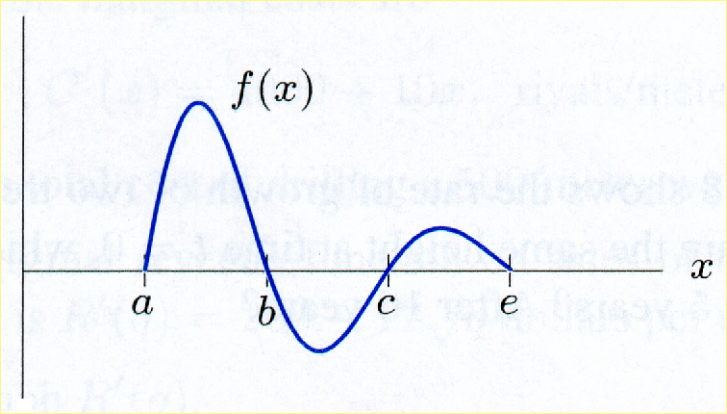
$$\int_1^3 z + \frac{1}{z}\, dz = ?$$

$$F(z) = \frac{1}{2} \cdot z^2 + \ln z$$

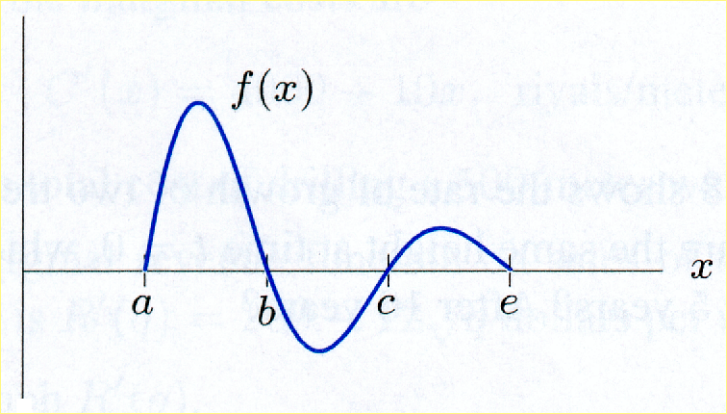
$$F(3) - F(1) = \left(\frac{1}{2} \cdot 3^2 + \ln 3\right) - \left(\frac{1}{2} \cdot 1^2 + \ln 1\right)$$

$$= \left(\frac{9}{2} + \ln 3\right) - \left(\frac{1}{2}\right) = 4 + \ln 3$$

Problem 17:



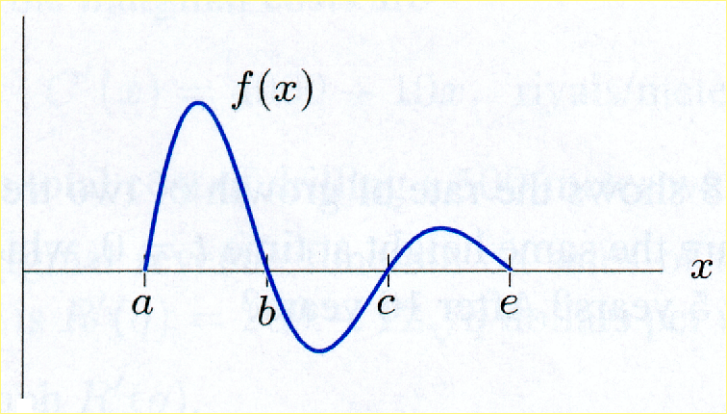
Problem 17:



I. $\int_a^b f(x)\,dx$ II. $\int_a^c f(x)\,dx$ III. $\int_a^e f(x)\,dx$

IV. $\int_b^e f(x)\,dx$ V. $\int_b^c f(x)\,dx$

Problem 17:



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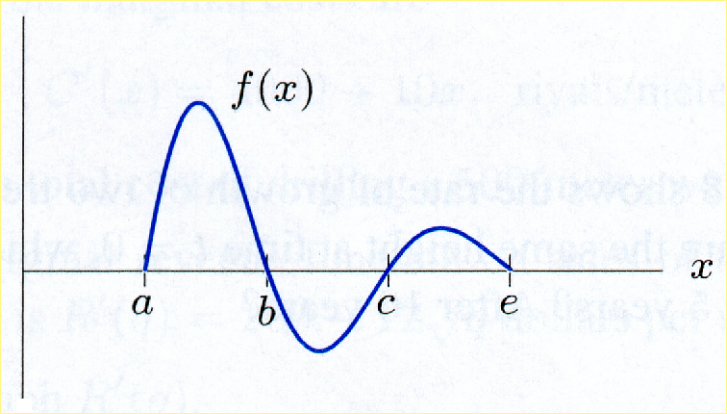
IV. $\int_b^e f(x)\,dx$ V. $\int_b^c f(x)\,dx$

Which are negative?

Problem 17:

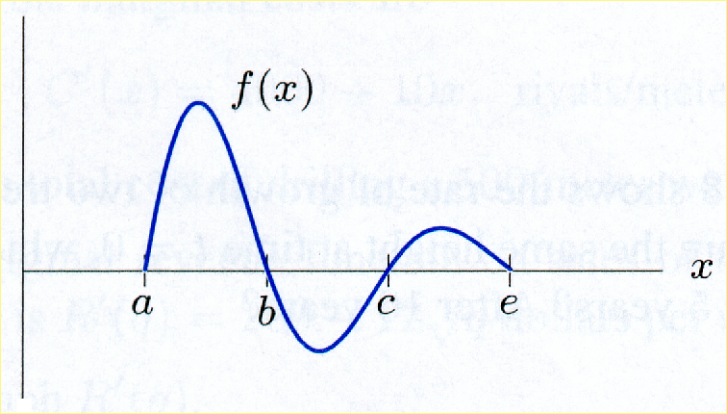


I. $\int_a^b f(x)\,dx$ II. $\int_a^c f(x)\,dx$ III. $\int_a^e f(x)\,dx$

IV. $\int_b^e f(x)\,dx$ V. $\int_b^c f(x)\,dx$

Which are negative? IV and V

Problem 17:

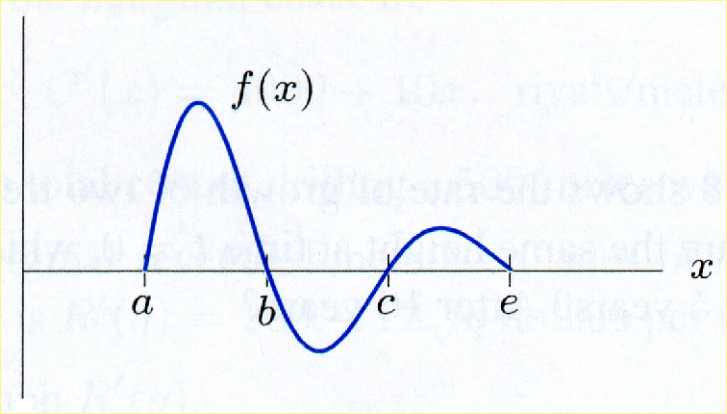


I. $\int_a^b f(x)\,dx$ II. $\int_a^c f(x)\,dx$ III. $\int_a^e f(x)\,dx$

IV. $\int_b^e f(x)\,dx$ V. $\int_b^c f(x)\,dx$

Write in increasing order.

Problem 17:



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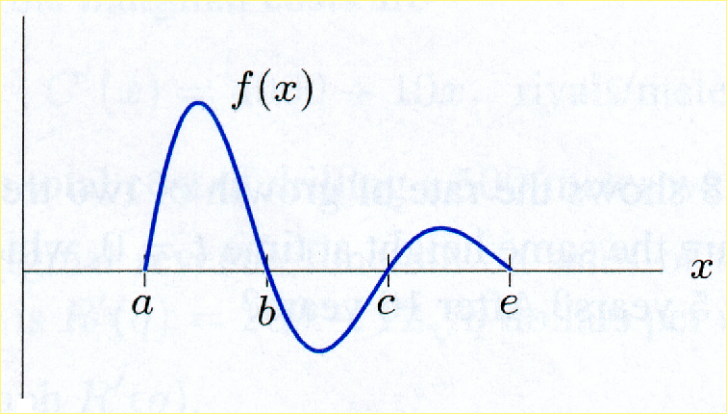
IV. $\int_b^e f(x)\,dx$ V. $\int_b^c f(x)\,dx$

Write in increasing order.

V,

Problem 17:



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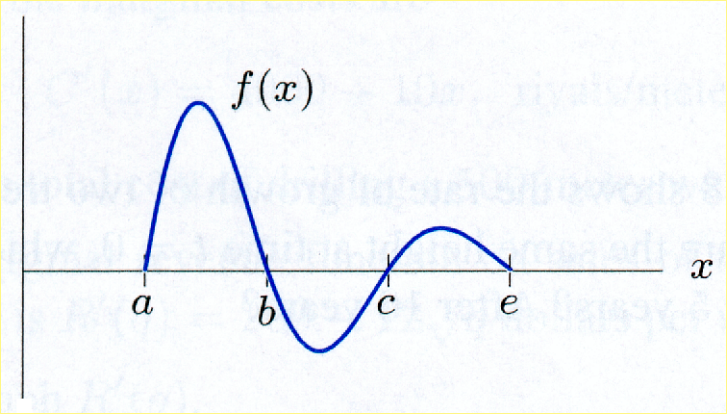
IV. $\int_b^e f(x)\,dx$ V. $\int_b^c f(x)\,dx$

Write in increasing order.

V, IV,

Problem 17:



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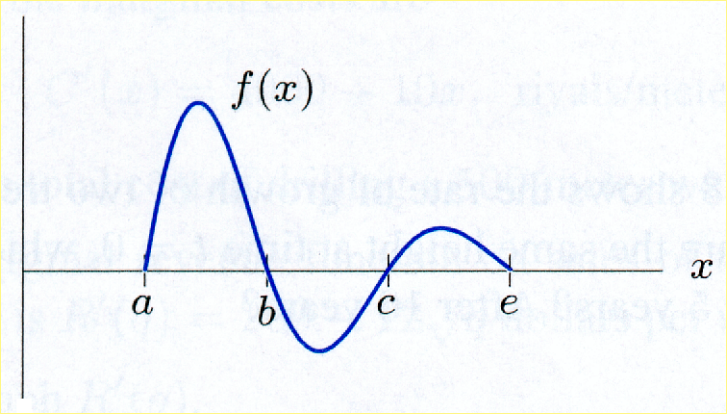
IV. $\int_b^e f(x)\,dx$ V. $\int_b^c f(x)\,dx$

Write in increasing order.

V, IV, II,

Problem 17:



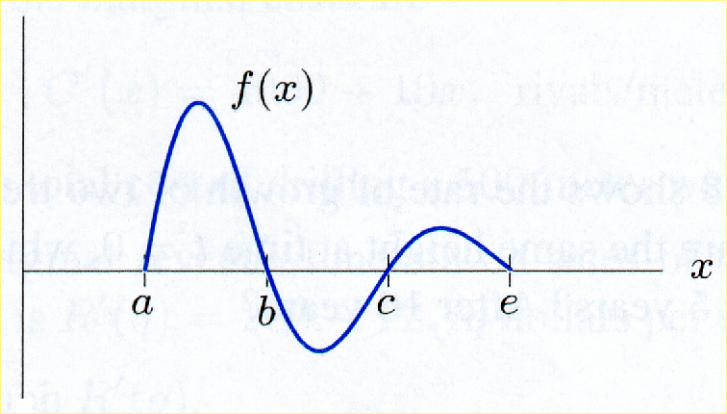
I. $\int_a^b f(x)\,dx$ II. $\int_a^c f(x)\,dx$ III. $\int_a^e f(x)\,dx$

IV. $\int_b^e f(x)\,dx$ V. $\int_b^c f(x)\,dx$

Write in increasing order.

V, IV, II, III,

Problem 17:



I. $\int_a^b f(x)\,dx$ II. $\int_a^c f(x)\,dx$ III. $\int_a^e f(x)\,dx$

IV. $\int_b^e f(x)\,dx$ V. $\int_b^c f(x)\,dx$

Write in increasing order.

V, IV, II, III, I

Problem 25 (b,c):

$$C'(q) = 0.005q^2 - q + 56$$

fixed costs are $20000

Problem 25 (b,c):

$$C'(q) = 0.005q^2 - q + 56$$

fixed costs are \$20000

$C(150) = ?$

Problem 25 (b,c):

$$C'(q) = 0.005q^2 - q + 56$$

fixed costs are \$20000

$$C(150) = C(150) - C(0) + 20000$$

Problem 25 (b,c):

$$C'(q) = 0.005q^2 - q + 56$$

fixed costs are \$20000

$$C(150) = C(150) - C(0) + 20000$$

$$= \int_0^{150} C'(q)\,dq + 20000$$

Problem 25 (b,c):

$$C'(q) = 0.005q^2 - q + 56$$

fixed costs are \$20000

$$C(150) = C(150) - C(0) + 20000$$

$$= \int_0^{150} C'(q)\,dq + 20000$$

Problem 25 (b,c):

$$\int_0^{150} (0.005q^2 - q + 56)\, dq$$

Problem 25 (b,c):

$$\int_0^{150} (0.005q^2 - q + 56)\, dq$$

$$F(q) = \frac{0.005q^3}{3} - \frac{q^2}{2} + 56q$$

Problem 25 (b,c):

$$\int_0^{150} (0.005q^2 - q + 56)\, dq$$

$$F(q) = \frac{0.005q^3}{3} - \frac{q^2}{2} + 56q$$

$$F(150) - F(0) = 2775$$

Problem 25 (b,c):

$$C'(q) = 0.005q^2 - q + 56$$

fixed costs are \$20000

$$C(150) = C(150) - C(0) + 20000$$

$$= \int_0^{150} C'(q)\,dq + 20000$$

Problem 25 (b,c):

$$C'(q) = 0.005q^2 - q + 56$$

fixed costs are \$20000

$$\begin{aligned} C(150) &= C(150) - C(0) + 20000 \\ &= \int_0^{150} C'(q)\,dq + 20000 \\ &= 2775 + 20000 \end{aligned}$$

Problem 25 (b,c):

$$C'(q) = 0.005q^2 - q + 56$$

fixed costs are \$20000

$$\begin{aligned} C(150) &= C(150) - C(0) + 20000 \\ &= \int_0^{150} C'(q)\,dq + 20000 \\ &= 2775 + 20000 = 22775 \end{aligned}$$

Problem 25 (b,c):

$$C'(q) = 0.005q^2 - q + 56$$

fixed costs are \$20000

$C'(150) = ?$

Problem 25 (b,c):

$$C'(q) = 0.005q^2 - q + 56$$

fixed costs are \$20000

$$C'(150) = 0.005 \cdot 150^2 - 150 + 56 = 18.5$$

Problem 25 (b,c):

$$C'(q) = 0.005q^2 - q + 56$$

fixed costs are \$20000

$$C'(150) = 0.005 \cdot 150^2 - 150 + 56 = 18.5$$

Interpretation: It will cost about \$18.50 to produce the 151^{st} item.