Math 580/780I Review Sheet for Test 2

Stuff to Know:

- The statement of Fermat's Little Theorem (Theorem 11).
- The definitions of a pseudoprime and of an absolute pseudoprime.
- The statement of Euler's Theorem (Theorem 12).
- The statement of Wilson's Theorem (Theorem 13).
- The Chinese Remainder Theorem (Theorem 14)!!
- How to do the quiz problems.
- How to do homework problems and examples from class. This includes items not mentioned above. See class presentations for examples.

Practice Problems:

1. Prove $1905 = 3 \cdot 5 \cdot 127$ is a pseudoprime.
2. Let a and m be relatively prime integers with $m > 0$. We showed that a has an inverse modulo m (see Theorem 10 of Notes 5). In other words, we showed that there is an integer x satisfying $ax \equiv 1 \pmod{m}$. Explain why Euler's Theorem implies that such an x exists. (Hint: Take x to be the "right" power of a.)
3. What is the remainder when $1234567891011\ldots4647$ is divided by 30?
4. Solve the system of congruences below. (Hint: $2^3 \cdot 3 \cdot 5 \cdot 19 = 2280$.)
$$\begin{aligned} x &\equiv 10 \pmod{24} \\ x &\equiv 18 \pmod{20} \\ x &\equiv 13 \pmod{15} \\ x &\equiv 3 \pmod{95} \end{aligned}$$
5. What are the first two positive integers n that satisfy $8|n$, $9|(n+1)$, $10|(n+2)$, $11|(n+3)$ and $12|(n+4)$?
6. What is the remainder when 3^{100} is divided by 45?
7. What is the remainder when 2^{200} is divided by 216? Note that 2 and 216 are not relatively prime.