Math 580/780I Review Sheet for 2019 Test 1

Proofs to Know:

- Be able to show $\sqrt{5}$ is irrational.
- Be able to show $\log_2 3$ is irrational.
- Be able to handle a problem like Homework Problem 3 of Notes 2.
- The proof of Theorem 9 from Notes 5.

Other Stuff to Know:

- The statement of the Division Algorithm.
- The statement of the Fundamental Theorem of Arithmetic.
- The Euclidean Algorithm. If I mention it, you should know what I am talking about.
- The statement of Fermat's Little Theorem (Theorem 11).
- The definitions of a pseudoprime and of an absolute pseudoprime.
- The statement of Euler's Theorem (Theorem 12).
- The statement of Wilson's Theorem (Theorem 13).
- How to do homework problems, quiz problems and examples from class. This includes items not mentioned above. See class presentations for examples.

Practice Problems:

1. Prove that $\log_6 4$ is irrational.
2. How many positive divisors does the number $2 \cdot 3^4 \cdot 5^2$ have?
3. Observe that if $a = 11$ and $b = 7$, then $\gcd(a, b) = 1$ and $\gcd(a + b, a - b) = \gcd(18, 4) = 2$. Prove that if instead a and b are integers with one of them even and one of them odd, then
$$\gcd(a, b) = \gcd(a + b, a - b).$$
4. What are the *two* rightmost digits in the expansion for 99^{2010}? Justify your answer.
5. For each part below, determine if there is an integer x satisfying the given congruence. If so, then determine such an x. If not, explain why no such x exists. (Hint: Think about the definition of $a \equiv b \pmod{n}$.)

   (a) $30x \equiv 5 \pmod{65}$ (b) $31x \equiv 5 \pmod{65}$

   (c) $30x \equiv 6 \pmod{65}$ (d) $31x \equiv 6 \pmod{65}$
6. Prove $1905 = 3 \cdot 5 \cdot 127$ is a pseudoprime.
7. Let a and m be relatively prime integers with $m > 0$. We showed that a has an inverse modulo m (see Theorem 10 of Notes 5). In other words, we showed that there is an integer x satisfying $ax \equiv 1 \pmod{m}$. Explain why Euler's Theorem implies that such an x exists. (Hint: Take x to be the "right" power of a.)