Math 580/780I Review Sheet for Test 1

Proofs to Know:

- Be able to show $\sqrt{5}$ is irrational.

- Be able to show $\log_2 3$ is irrational.

- Be able to handle a problem like Homework Problem 3 of Notes 2.

- The proof of Theorem 9 from Notes 5.

Other Stuff to Know:

- The statement of the Division Algorithm.

- The statement of the Fundamental Theorem of Arithmetic.

- The Euclidean Algorithm. If I mention it, you should know what I am talking about.

- Homework problems and examples from class. This includes items not mentioned above. See class presentations for examples.

Practice Problems:

1. Prove that $\log_6 4$ is irrational.

2. How many positive divisors does the number $2 \cdot 3^4 \cdot 5^2$ have?

3. Observe that if $a = 11$ and $b = 7$, then $\gcd(a, b) = 1$ and $\gcd(a + b, a - b) = \gcd(18, 4) = 2$. Prove that if instead a and b are integers with one of them even and one of them odd, then

$$\gcd(a, b) = \gcd(a + b, a - b).$$

4. What are the *two* rightmost digits in the expansion for 99^{2010}? Justify your answer.

5. For each part below, determine if there is an integer x satisfying the given congruence. If so, then determine such an x. If not, explain why no such x exists. (Hint: Think about the definition of $a \equiv b \pmod{n}$.)

   (a) $30x \equiv 5 \pmod{65}$ (b) $31x \equiv 5 \pmod{65}$

   (c) $30x \equiv 6 \pmod{65}$ (d) $31x \equiv 6 \pmod{65}$