Math 580/780I: Test 1, Fall 2010

Instructions and Point Values: Put your name at the top of the first page of the blank paper given to you. There are 7 problems below. Work each problem on the paper provided, using a separate page for each problem. Show ALL of your work. Do NOT use a calculator.

There are 100 total points possible on this exam. The point value for each problem appears to the left of each problem.

14 pts (1) Prove that $\log_4 7$ is an irrational number. Use complete English sentences.

14 pts (2) Calculate $\gcd(25979, 2573)$ using the Euclidean algorithm. Show your work and circle the value of the greatest common divisor.

14 pts (3) Find integers x and y such that $25979x + 2573y = \gcd(25979, 2573)$. You may want to use your calculations from the previous problem. Circle the values of x and y and be sure to clarify which value is x and which value is y.

14 pts (4) Observe that $(x, y) = (2, -2)$ is a solution to the equation $18x + 15y = 6$. Describe ALL the pairs of integers (x, y) that satisfy the equation $18x + 15y = 6$. Clarify whether any parameters in your answer are intended to be integers, positive integers, rational numbers, etc. Circle your answer.

14 pts (5) Observe that $2^4 \equiv 16 \equiv 1 \pmod{5}$ and $3^4 \equiv 81 \equiv 1 \pmod{5}$. Also, $2010 = 4 \cdot 502 + 2$. Using this information, calculate the remainder when $2^{2010} + 3^{2010}$ is divided by 5. Show the work justifying your answer and circle it.

14 pts (6) Let a, b and d be integers. Using the definition of what it means for one integer to divide another, prove that if d divides a and d divides b, then d divides $2a - 3b$. The proof should be short, but use complete English sentences and make your explanation clear.

16 pts (7) For each part below, determine if there is an integer x satisfying the given congruence. If so, then find a *positive* integer x satisfying the congruence and circle it. If not, explain why no integer x exists. To possibly help you with this problem, I have indicated the complete prime factorization of numbers occurring in this problem and some Euclidean algorithm computations associated with numbers in this problem. You do not need to check this information and can feel free to use it.

(a) $2010\,x \equiv 2 \pmod{162}$

(b) $2010\,x \equiv 2 \pmod{163}$

"Possible" Useful Information

$2010 = 2 \cdot 3 \cdot 5 \cdot 67$	$2010 = 162 \cdot 12 + 66$	$2010 = 163 \cdot 12 + 54$
$162 = 2 \cdot 3^4$	$162 = 66 \cdot 2 + 30$	$163 = 54 \cdot 3 + 1$
$163 = 163$ (it's a prime)	$66 = 30 \cdot 2 + 6$	$54 = 1 \cdot 54 + 0$
	$30 = 6 \cdot 5 + 0$	