

Math 580: Quiz 8

Show ALL Work

Name _____ **Solutions** _____

1. Calculate $\gcd(x^7 + x^6 + x^5 + 3x^3 + x^2 - 2, x^4 + x^3 + x^2 + 2)$.

Answer:

$$x^2 + 2x + 2$$

Solution. By the Euclidean algorithm, the computations

$$x^7 + x^6 + x^5 + 3x^3 + x^2 - 2 = (x^4 + x^3 + x^2 + 2)x^3 + x^3 + x^2 - 2$$

$$x^4 + x^3 + x^2 + 2 = (x^3 + x^2 - 2)x + x^2 + 2x + 2$$

$$x^3 + x^2 - 2 = (x^2 + 2x + 2)(x - 1) + 0$$

imply that the answer is $x^2 + 2x + 2$. ■

2. Using your computations above, find $u(x)$ and $v(x)$ satisfying

$$\begin{aligned} (x^7 + x^6 + x^5 + 3x^3 + x^2 - 2)u(x) + (x^4 + x^3 + x^2 + 2)v(x) \\ = \gcd(x^7 + x^6 + x^5 + 3x^3 + x^2 - 2, x^4 + x^3 + x^2 + 2). \end{aligned}$$

$$u(x) =$$

$$-x$$

$$v(x) =$$

$$x^4 + 1$$

Solution. From the work in the previous problem, we see that

$$\begin{aligned} x^2 + 2x + 2 &= (x^4 + x^3 + x^2 + 2) - (x^3 + x^2 - 2)x \\ &= (x^4 + x^3 + x^2 + 2) - ((x^7 + x^6 + x^5 + 3x^3 + x^2 - 2) - (x^4 + x^3 + x^2 + 2)x^3)x \\ &= (x^7 + x^6 + x^5 + 3x^3 + x^2 - 2)(-x) + (x^4 + x^3 + x^2 + 2)(x^4 + 1), \end{aligned}$$

giving the answers indicated above. ■