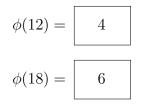
## Math 580: Quiz 5

## Show ALL Work Name Solutions

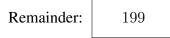
1. Calculate  $\phi(12)$  and  $\phi(18)$ .



**Solution.** The only positive integers relatively prime to 12 and  $\leq 12$  are 1, 5, 7 and 11. Therefore,  $\phi(12) = 4$ . The only positive integers relatively prime to 18 and  $\leq 18$  are 1, 5, 7, 11, 13 and 17. Therefore,  $\phi(18) = 6$ .  $\Box$ 

**Curiosity.** Despite appearances, most numbers n do not have only 1 and primes  $\leq n$  that are relatively prime to n. In fact, the largest number n that has only 1 and primes  $\leq n$  that are relatively prime to n is n = 30. The numbers  $\leq 30$  that are relatively prime to 30 are 1, 7, 11, 13, 17, 19, 23 and 29. But don't expect 1 and primes in general when looking at numbers relatively prime to n and less than or equal to n.

2. Given that  $\phi(825) = 400$ , what is the remainder when  $2^{10012010}$  is divided by 825? Note that the remainder should be in the set  $\{0, 1, \dots, 824\}$ .



**Solution.** Since 2 and 825 are relatively prime, we can use Euler's Theorem to get  $2^{400} \equiv 1 \pmod{825}$ . Dividing, we see that  $10012010 = 400 \cdot 25030 + 10$ . Hence,

 $2^{10012010} \equiv \left(2^{400}\right)^{25030} \cdot 2^{10} \equiv 1^{25030} \cdot 2^{10} \equiv 2^{10} \equiv 1024 \equiv 199 \pmod{825}.$ 

Hence, the remainder is 199.  $\Box$