Math 580: Quiz 5

Show ALL Work Name **Solutions**

1. Calculate $\phi(12)$ and $\phi(18)$.

   $\phi(12) =$ 4

   $\phi(18) =$ 6

   Solution. The only positive integers relatively prime to 12 and $\leq$ 12 are 1, 5, 7 and 11. Therefore, $\phi(12) = 4$. The only positive integers relatively prime to 18 and $\leq$ 18 are 1, 5, 7, 11, 13 and 17. Therefore, $\phi(18) = 6$. $\square$

   Curiosity. Despite appearances, most numbers n do not have only 1 and primes $\leq n$ that are relatively prime to n. In fact, the largest number n that has only 1 and primes $\leq n$ that are relatively prime to n is $n = 30$. The numbers ≤ 30 that are relatively prime to 30 are 1, 7, 11, 13, 17, 19, 23 and 29. But don't expect 1 and primes in general when looking at numbers relatively prime to n and less than or equal to n.

2. Given that $\phi(825) = 400$, what is the remainder when $2^{10012010}$ is divided by 825? Note that the remainder should be in the set $\{0, 1, \ldots, 824\}$.

   Remainder: 199

   Solution. Since 2 and 825 are relatively prime, we can use Euler's Theorem to get $2^{400} \equiv 1 \pmod{825}$. Dividing, we see that $10012010 = 400 \cdot 25030 + 10$. Hence,

   $$2^{10012010} \equiv \left(2^{400}\right)^{25030} \cdot 2^{10} \equiv 1^{25030} \cdot 2^{10} \equiv 2^{10} \equiv 1024 \equiv 199 \pmod{825}.$$

   Hence, the remainder is 199. $\square$