# Math 580: Quiz 4

**Show ALL Work** Name **Solutions**

1. The number $2017$ is a prime. What is the smallest positive integer $m$ such that

$$2019^{2018} \equiv m \pmod{2017}\,?$$

Show the work leading to your answer.

$m = \boxed{4}$

**Solution.** By Fermat's Little Theorem, we have $2019^{2016} \equiv 1 \pmod{2017}$. Thus,

$$2019^{2018} \equiv 2019^{2012} \cdot 2^2 \equiv 1 \cdot 2^2 \equiv 4 \pmod{2017}. \quad \square$$

2. The prime factorization of $5461$ and $5460$ are given by

$$5461 = 43 \cdot 127 \qquad \text{and} \qquad 5460 = 2^2 \cdot 3 \cdot 5 \cdot 7 \cdot 13.$$

Prove that $5461$ is a pseudoprime. Use English sentences throughout your proof.

**Solution.** By Fermat's Little Theorem, we have $2^{42} \equiv 1 \pmod{43}$ and $2^{126} \equiv 1 \pmod{127}$. Since $5460 = 42 \cdot 130$, we deduce

$$2^{5460} \equiv \left(2^{42}\right)^{130} \equiv 1^{130} \equiv 1 \pmod{43}.$$

On the other hand, $126$ does not divide $5460$, so we have to do more work for the prime $127$. On the other other hand, we have $2^7 = 128 \equiv 1 \pmod{127}$. Since $5460 = 7 \cdot 780$, we deduce

$$2^{5460} \equiv \left(2^{7}\right)^{780} \equiv 1^{780} \equiv 1 \pmod{127}.$$

Hence, $2^{5460} - 1$ is divisible by $43$ and $127$. Since $43$ and $127$ are relatively prime, we deduce that $2^{5460} - 1$ is divisible by $43 \cdot 127 = 5461$ . Therefore,

$$2^{5460} \equiv 1 \pmod{5461} \quad \Longrightarrow \quad 2^{5461} \equiv 2 \pmod{5461},$$

establishing that $5461$ is a pseudoprime. $\square$