MATH 574, NOTES 5
COUNTING BASICS

Examples:

(1) How many different three letter initials can people have?

(2) At a certain university, a committee consisting currently of six adminstrative officials is looking for another member, either among its 2000 faculty or among its 19000 students. How many possibilities are there for the additional committee member?

(3) How many five digit numbers are there?

(4) Two dice are rolled, one is red and the other is green, so that two numbers between one and six land face-up. How many possibilities are there for the outcome? What if the two dice were identical (both the same color and size)?

(5) How many subsets does an n-element set have?

(6) Each person in a group of ten people has two coins. How many coins are there all together?

(7) A palindromic number is a number which reads the same way forward and backward, like 411626114. How many five digit palindromic numbers are there?

(8) Consider the positive integers ≤ 100. How many are even? How many are divisible by 5? How many are divisible by 3? How many are divisible by either 2 or 5 (possibly both)? How many are divisible by 2, 3, or 5?

(9) In how many zeroes does the number 100! end?

(10) How many different ways can the letters a, b, c, and d be lined up if each letter is to be used exactly once? What if we add the restriction that the letter b does not come immediately before the letter c? What if the letter b must come after the letter c but not necessarily immediately after?

(11) Five cards are selected from a deck of 52 cards to form a poker hand. How many poker hands are possible?

(12) Suppose D is a set containing n elements and S is a set containing k elements. How many different functions f are there with domain D such that $f(x) \in S$ for every $x \in D$ (in other words, how many different functions are there mapping D into S)? How many of these are one-to-one functions (so that $f(x) \neq f(y)$ whenever $x \neq y$)? How many are one-to-one and onto? (What are the restrictions needed on n and k in each case?)