

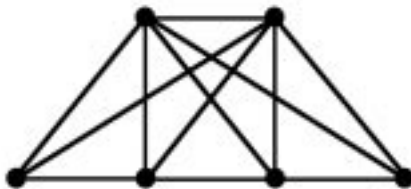
MATH 574, NOTES 11
PRACTICE PROBLEMS FOR TEST 3

- (1) Suppose $a_0 = 2$, $a_1 = 3$, and $a_n = 3a_{n-1} - 2a_{n-2}$ for $n \geq 2$. Find an explicit formula for a_n in terms of n .
- (2) Suppose $u_0 = 4$, $u_1 = 4$, $u_2 = 6$, and $u_n = 2u_{n-1} - u_{n-2} + 2u_{n-3}$ for $n \geq 3$. Find an explicit formula for u_n in terms of n .
- (3) Let S_n denote the number of ways to cover the squares of a $2 \times n$ board using any number of 1×2 pieces, 2×1 pieces, and 2×2 pieces (without overlapping the pieces). For example, $S_1 = 1$, $S_2 = 3$, and $S_3 = 5$. Find an explicit formula for S_n in terms of n .
- (4) (a) Let $G = (V, E)$ be a graph. Prove that

$$\sum_{v \in V} \deg(v) = 2|E|.$$

In other words, explain why the sum of all the degrees of the vertices is equal to twice the number of edges in a graph.

- (b) Let n be an integer ≥ 2 . Explain why (a) and the fact that a tree on n vertices has $n - 1$ edges imply that a tree on n vertices has at least two vertices with degree 1.
- (5) Is the graph below planar? Justify your answer.



- (6) Two people play a game. We begin with $N = 0$. Each person takes turns choosing a number from $\{1, 2, 3, 4\}$, adding it to N to form a new number N , and announcing what the new N is. The winner is the first person to get the number N to be ≥ 100 .
- (a) Is it better to move first or second in this game? (Don't answer, "Yes.")
- (b) If the first player begins by choosing the number 4, what is the best number for the second player to choose from the set $\{1, 2, 3, 4\}$?