

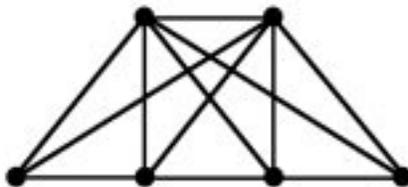
**MATH 574, NOTES 11**  
**PRACTICE PROBLEMS FOR TEST 3**

- (1) Suppose  $a_0 = 2$ ,  $a_1 = 3$ , and  $a_n = 3a_{n-1} - 2a_{n-2}$  for  $n \geq 2$ . Find an explicit formula for  $a_n$  in terms of  $n$ .
- (2) Suppose  $u_0 = 4$ ,  $u_1 = 4$ ,  $u_2 = 6$ , and  $u_n = 2u_{n-1} - u_{n-2} + 2u_{n-3}$  for  $n \geq 3$ . Find an explicit formula for  $u_n$  in terms of  $n$ .
- (3) Let  $S_n$  denote the number of ways to cover the squares of a  $2 \times n$  board using any number of  $1 \times 2$  pieces,  $2 \times 1$  pieces, and  $2 \times 2$  pieces (without overlapping the pieces). For example,  $S_1 = 1$ ,  $S_2 = 3$ , and  $S_3 = 5$ . Find an explicit formula for  $S_n$  in terms of  $n$ .
- (4) (a) Let  $G = (V, E)$  be a graph. Prove that

$$\sum_{v \in V} \deg(v) = 2|E|.$$

In other words, explain why the sum of all the degrees of the vertices is equal to twice the number of edges in a graph.

- (b) Let  $n$  be an integer  $\geq 2$ . Explain why (a) and the fact that a tree on  $n$  vertices has  $n - 1$  edges imply that a tree on  $n$  vertices has at least two vertices with degree 1.
- (5) Is the graph below planar? Justify your answer.



- (6) Two people play a game. We begin with  $N = 0$ . Each person takes turns choosing a number from  $\{1, 2, 3, 4\}$ , adding it to  $N$  to form a new number  $N$ , and announcing what the new  $N$  is. The winner is the first person to get the number  $N$  to be  $\geq 100$ .
- (a) Is it better to move first or second in this game? (Don't answer, "Yes.")
- (b) If the first player begins by choosing the number 4, what is the best number for the second player to choose from the set  $\{1, 2, 3, 4\}$ ?