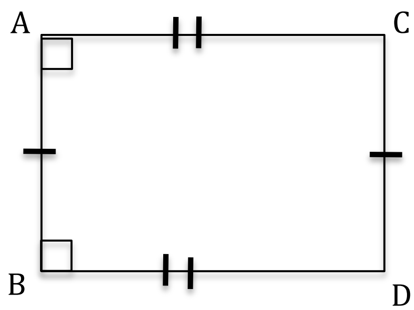
1. Using Theorem 1 with t = 1/10 we see that C = (1-t) A + t B
2. Using the rules we get a shape that looks like this!



$$(A-B)^2 = (C-D)^2 \rightarrow |BA| = |DC|$$
$$(A-C)^2 = (B-D)^2 \longrightarrow |CA| = |DB|$$
$$(A-B)(A-C) = 0 \longrightarrow \overline{BA} \text{ is perpendicular to } \overline{CA}$$
$$(B-A)(B-D) = 0 \rightarrow \overline{AB} \text{ is perpendicular to } \overline{DB}$$

So. iii is true because we have a four sided shape where the opposite sides are the same length and the interior angles are all 90 degrees deduced from the orthogonality of BA to CA and AB to DB.

3. $N = \frac{A+B+C+D}{4}, M_A = \frac{B+C}{2}, M_B = \frac{A+C}{2}$

$$\text{To show that } |NM_A| = |NM_B| \text{ we'llshow that } (N-M_A)^2 = (N-M_B)^2$$

$$(N-M_A)^2 = (N-M_B)^2$$
$$\left(\frac{A+B+C+D}{4} - \frac{B+C}{2}\right) = \left(\frac{A+B+C+D}{4} - \frac{A+C}{2}\right)$$
$$\left(\frac{A-B-C+D}{4}\right)^2 = \left(\frac{-A+B-C+D}{4}\right)^2$$
$$A^2+B^2+C^2+D^2-2AB-2AC+2AD+2BC-2BD-2CD =$$
$$A^2+B^2+C^2+D^2-2AB+2AC-2AD-2BC+2BD-2CD$$

From this we get

$$AC - AD - BC + BD = 0$$
$$A(C-D) - B(C-D) = 0$$
$$(A-B)(C-D) = 0$$

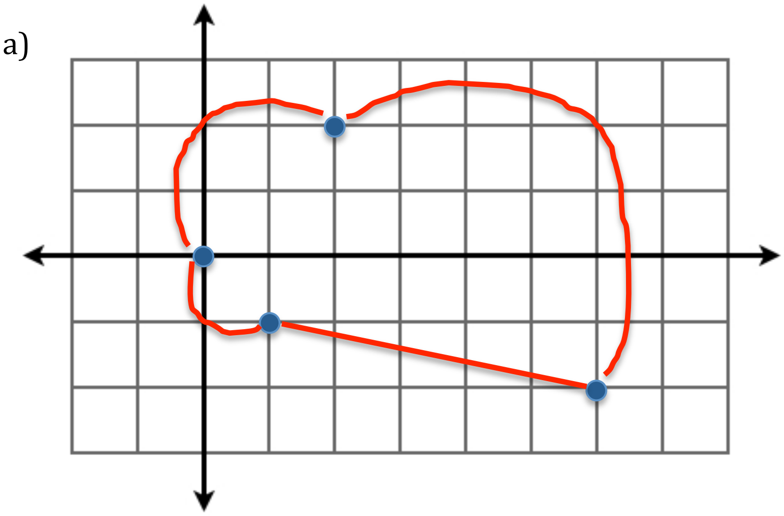
We know that $\overline{BA}$ is perpendicular to $\overline{DC}$. This means that the last equation above is true. Thus, $(N-M_A)^2 = (N-M_B)^2$ and $|NM_A| = |NM_B|$ can be shown by reversing the steps above.

4.

- We first explain why $k_1 \neq 1$. If $k_1 = 1$ then the vectors AB and A'B' would be equal so the lines AB and A'B' would not intersect at P giving a contradiction.
- $t = -k_1/(1-k_1)$
- $(k_2 - 1)R = k_2C - A$
- $k_1 - k_2 = 0 \longrightarrow k_1 = k_2$
- $(1-k_1)P + (k_2-1)R = k_2C - k_1B$
- $(1-k_1) * (P-R) = k_1 * (C-B)$
- We deduce that the vector RP and the vector BC are parallel. Hence, lines PR and BC are parallel, completing the proof.

5. $f = R_{\pi,(4,0)} T_{(5,-1)} R_{\left(\frac{\pi}{2}\right),(1,0)} R_{\pi,(1,1)}$

a)

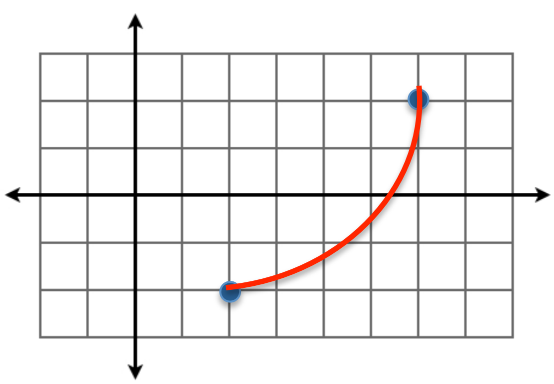


$f(2,2) = (2,2)$

b) $f = R_{\left(\frac{\pi}{2}\right),(2,2)}$ *from using Theorem 4*

c) Calculate the value of (2,-2) from b.

$f(2,-2) = (6,2)$



6.

a. $f(B) = E$

b. P = H. By theorem 4 we know that $f = R_{\pi,P}$ and we also know that $f(B) = E$ so $f(B) = R_{\pi,P}(B) = E$ for some point P. Looking at the picture we see that P = H taking B through a rotation of π around H gets us to E as desired.

c. Note that $f(A) = R_{\frac{\pi}{3},C} R_{\frac{\pi}{3},E} R_{\frac{\pi}{3},A}(A) = R_{\frac{\pi}{3},C} R_{\frac{\pi}{3},E}(A) = R_{\frac{\pi}{3},C}(F) = G$. Also, we see that $f(A)$ *is* $R_{\pi,H}(A)$ and it follows that H is on line AG and $|AH| = |HG|$. Hence, H is the midpoint of AG.