Solutions for Test 2

(1) Answer: $(B-A)(C-B)=0$

(2) For the rotation, we find that $f=\begin{pmatrix} -1 & 0 & 2C_x \\ 0 & -1 & 2C_y \\ 0 & 0 & 1 \end{pmatrix}$. If we apply this to $f(A)=\boldsymbol{B}$ then we can say: $\begin{pmatrix} -1 & 0 & 2C_x \\ 0 & -1 & 2C_y \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 8 \\ 1 \end{pmatrix}=\begin{pmatrix} -2012 \\ 2012 \\ 1 \end{pmatrix}$.
Multiplication of the matrices gives us $-2+2C_x=-2012$ and $-8+2C_y=2012$, from which we can find that $C_x=-1005$ and $C_y=1010$, so $C=(-1005,1010)$

(3) We are given

$$N=\frac{A+B+C+D}{4},\ M_A=\frac{B+C}{2},\ M_B=\frac{A+C}{2} \tag{1}$$

We want to show that $|NM_A|=|NM_B|$. We can do this by showing that $(N-M_A)^2=(N-M_B)^2$. This gives us
$(\frac{A+B+C+D}{4}-\frac{B+C}{2})^2=(\frac{A+B+C+D}{4}-\frac{A+C}{2})^2$

This simplifies to
$A^2+B^2+C^2+D^2-2AB-2AC+2AD+2BC-2BD-2CD=$
$A^2+B^2+C^2+D^2-2AB+2AC-2AD-2BC+2BD-2CD$
which gives us $AC-AD+BD-BC=0$.

Using this we get $A(C-D)-B(C-D)=0$ and $(A-B)(C-D)=0$. Since we know that AB and CD are perpendicular this must be true. $\therefore$ $|NM_A|=|NM_B|$.

#4

- $K_1 \neq 1$
- Give explanation here: If $k_1 = 1$ ~~Line~~ vector BA = ~~line~~ vector B'A' which implies line BA and line B'A' are parallel which contradicts that P exists.
- $t = -k_1/1-k_1$
- $(k_1-k_2)Q=k_1B-K_2C$
- $(k_2-1)R=k_2C-A$
- 2

#5 $R_{\pi,(-1,13)}$

$$f = R_{\frac{\pi}{2},(-1,6)}\, T_{(20,12)}\, R_{\frac{\pi}{2}(2,-3)}$$

$$f = R_{\pi,(-1,13)}$$

$$\begin{pmatrix} 0 & -1 & 5 \\ 1 & 0 & 7 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 20 \\ 0 & 1 & 12 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & -5 \\ 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & -1 & 5 \\ 1 & 0 & 7 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 0 & -1 & 19 \\ 1 & 0 & 7 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\downarrow$$

$$\begin{pmatrix} -1 & 0 & -2 \\ 0 & -1 & 26 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} -x-2 \\ -y+26 \\ 1 \end{pmatrix}$$

$$-x-2=x \rightarrow -2=2x$$
$$-y+26=y \rightarrow 26=2y$$

$$x=-1$$
$$y=13$$

#6

a. f(A)=A
b. P=A
c. $F(M_B)=D$
d. $R_{\pi/3,E}(R_{\pi,M_B}(M_B))=F$
e. $R_{\pi,M_c}(F)=D$ explanation: $R_{\pi,M_C}\, R_{\pi/3,E}\, R_{\pi,M_B}(M_B)=D$ by part (c). From part (d), we get $D = R_{\pi,M_c}(R_{\pi/3,E}\, R_{\pi,M_B}(M_B))$ ~~$R_{\pi/3,E}\, R_{\pi,M_B}(M_B)=f$~~ $= R_{\pi,M_c}(F)$.
f. Part E tells you that D is the π rotation about M_c, so the M_c must be the midpoint and that they must be collinear.