MATH 532, 736I: MODERN GEOMETRY

Test 1, Spring 2013 Name ____________________________

Show All Work

Instructions: This test consists of 3 pages of problems. Put your name at the top of this page and at the top of the first page of the packet of blank paper given to you. Work each problem in the packet of paper unless it is indicated that you can or are to do the work below. Show <u>ALL</u> of your work. Do <u>NOT</u> use a calculator.

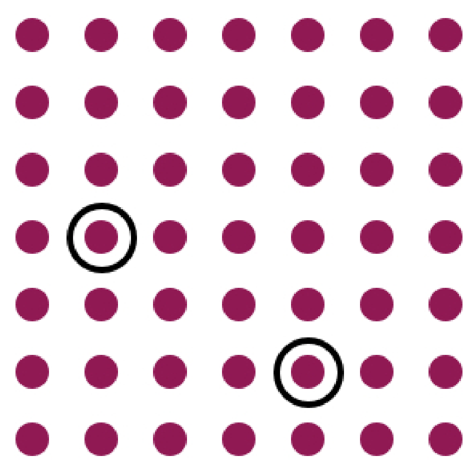
Points: Part I (53 pts), Part II (47 pts)

Part I. The point value for each problem appears to the left of each problem. In Problem 5, I will assume you are using the axioms as you state them in your answer to Problem 1 below.

8 pts (1) In the packet of white paper provided to you, state the axioms for a finite AFFINE plane of order n. (Number or name the axioms so you can refer to them in Problem 5.)

10 pts (2) Draw a model for a finite PROJECTIVE plane of order 3. Be sure to clearly mark every point and clearly draw every line in your model. If I look at your model, I should be able to tell where each of your points and lines are. In particular, make sure that each line you draw cannot be mistaken for two lines.

8 pts (3) Two points have been circled in the 7×7 array of points to the right. Using the model for a finite affine plane of order 7 discussed in class, finish circling the points that belong to the same line as the given circled points. (You do not need to use the packet of blank paper for this problem.)



12 pts (4) Consider the points $(10, 41)$ and $(30, 26)$ in a 59×59 array of points for our model of a finite affine plane of order 59. Find the equation of the line passing through these two points. Put your answer below in the form $y \equiv mx + k \pmod{59}$ where m and k are among the numbers $0, 1, 2, \ldots, 58$. Be sure to show your work in the packet provided with this test. You will not get credit for a correct answer without correct work.

Answer: $\boxed{}$

15 pts (5) Using only the theorem below and the axioms you stated in Problem 1, fill in the boxes below to complete a proof that in an affine plane of order n, for each line ℓ, there are *at least* $n-1$ lines parallel to ℓ. This is a proof you were to have memorized for class.

Theorem. *In an affine plane of order n, each line contains exactly n points.*

Note: The theorem is to be used in the proof below. The proof is establishing that there are at least $n-1$ lines parallel to ℓ as stated above.

Proof. Let ℓ be an arbitrary line. By $\boxed{}$, there is a point P_1 not on ℓ. By $\boxed{}$, line ℓ has at least one point, say P_2, on it. From $\boxed{}$, there is a line ℓ' passing through P_1 and P_2. Since $\boxed{}$, we have that $\ell' \neq \ell$. $\boxed{}$ implies that P_2 is the only point on both ℓ' and ℓ. By $\boxed{}$, ℓ' has exactly $\boxed{}$ points on it, two of which are P_1 and P_2. Let $P_3, \ldots, P_n$ denote the remaining points on ℓ'. For each $j \neq 2$, P_j is not on ℓ so that $\boxed{}$ implies that there is a line ℓ_j passing through P_j and parallel to ℓ. Each such ℓ_j is different from ℓ' since

$\boxed{}$.

It follows that the lines ℓ_j (with $j \neq 2$) are distinct by $\boxed{}$ (since ℓ' is the unique line passing through any two of the P_j's). Thus, there are at least $n-1$ distinct lines parallel to ℓ (namely, the lines ℓ_j with $j \neq 2$). ∎

Part II. The point values appear to the left of each problem. The problems in this section all deal with an axiomatic system consisting of the following axioms.

Axiom 1. There exist 3 distinct noncollinear points.

Axiom 2. There exist 3 distinct collinear points.

Axiom 3. Given any 2 distinct points, there is exactly 1 line passing through them.

$\boxed{\text{8 pts}}$ (1) Justify that the axiomatic system is consistent.

$\boxed{\text{6 pts}}$ (2) Justify that the axiomatic system is *not* complete. Include some brief explanation, in complete English sentences, for your answer.

$\boxed{\text{12 pts}}$ (3) Justify that the axiomatic system is independent.

$\boxed{\text{6 pts}}$ (4) What is the dual of Axiom 2? Use correct English.

$\boxed{\text{15 pts}}$ (5) Fill in the boxes below to finish the proof below that, in the axiomatic system above, there are at least 4 distinct lines.

Proof. By $\boxed{}$, there is a line ℓ with at least 3 points on it. Call three such points P_1, P_2 and P_3. By $\boxed{}$, there is at least one point Q not on ℓ. By $\boxed{}$, there is exactly one line ℓ_j passing through Q and P_j for each $j \in \{1, 2, 3\}$. Each ℓ_j is not equal to ℓ because

$\boxed{}$.

Assume $\ell_i = \ell_j$ for some $i \neq j$ with i and j in $\{1, 2, 3\}$. Then $\boxed{}$ and $\boxed{}$ are two distinct points on the two distinct lines $\boxed{}$ and $\boxed{}$. This contradicts $\boxed{}$. Hence, the lines ℓ_1, ℓ_2 and ℓ_3 are all different. Since we have shown ℓ, ℓ_1, ℓ_2 and ℓ_3 are all different, the proof is complete. ■