MATH 532, 736I: MODERN GEOMETRY

Name ______________________________

Test #1

Show All Work
Points: Part I (60 pts), Part II (40 pts)

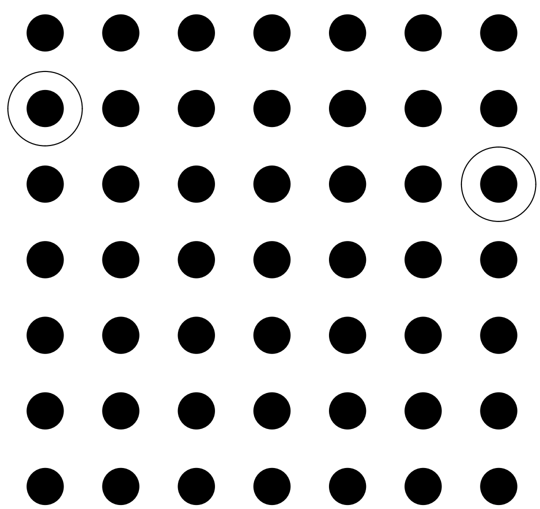
Part I. Each problem in this section is worth 10 points. The last problem, Problem 6, is one of the proofs that you were to have memorized. In Problem 6, I will assume you are using the axioms as you state them in your answer to Problem 2 below.

(1) State the axioms for a finite projective plane of order n.

(2) State the axioms for a finite affine plane of order n. (Number or name the axioms so you can refer to them.)

(3) Give a model for a finite projective plane of order 2. Be sure to clearly mark every point and clearly draw every line in your model.

(4) Two points have been circled in the 7×7 array of points below. Using the model for a finite affine plane of order 7 discussed in class, finish circling the points that belong to the same line as the given circled points.



(5) Consider the points $(6, 1)$ and $(12, 12)$ in a 17×17 array of points for our model of a finite affine plane of order 17. Find the equation of the line passing through these two points. Put your answer in the form $y \equiv mx + k \pmod{17}$ where m and k are among the numbers $0, 1, 2, \ldots, 16$.

(6) Using only the axioms you stated in Problem 2 (and referring to them whenever you use them), prove that if ℓ is a line in an affine plane of order n with exactly n points on it and A is a point not on ℓ, then there are exactly $n + 1$ lines passing through A.

Part II. The problems in this section all deal with an axiomatic system consisting of the axioms below. Be sure to answer the questions being asked. For example, if you are giving a model to justify your answer in Problem 1 below, make sure you also state whether your answer is, “Yes” or “No.” Problems 1 and 3 are worth 5 points each, Problem 2 is worth 9 points, Problem 4 is worth 6 points, and Problem 5 is worth 15 points.

Axiom 1. There exist at least 2 distinct lines.

Axiom 2. Every line contains at least 3 distinct points.

Axiom 3. Any two distinct lines do not intersect in more than one point.

(1) Is the axiomatic system consistent? Justify your answer.

(2) Is the axiomatic system independent? Justify your answer.

(3) Is the axiomatic system complete? Justify your answer.

(4) Does the Principle of Duality hold for this axiomatic system? Justify your answer.

(5) Prove that in this axiomatic system there exist at least 4 distinct points no 3 of which are collinear.