MATH 532, 736I: MODERN GEOMETRY

Name _____

Test #1

Show All Work
Points: Part I (34 pts), Part II (28 pts), Part III (38 pts)

Part I. Each of the following is something or part of something you were asked to memorize. In the proofs you give below, I will assume you are using the axioms as you state them in your answers to (1) and (2). The first 2 problems in this section are worth 7 points each, and the second 2 problems are worth 10 points each.

(1) State the axioms for a finite projective plane of order n. (Number or name the axioms so you can refer to them.)

(2) State the axioms for a finite affine plane of order n. (Number or name the axioms so you can refer to them.)

(3) Prove that in a finite projective plane of order n, there exist exactly $n^2 + n + 1$ points and exactly $n^2 + n + 1$ lines. For the proof, you should only use the axioms you stated in (1) and the two results below.

Theorem 1: *In a projective plane of order n, each point is incident with exactly $n + 1$ lines.*

Theorem 2: *In a projective plane of order n, each line is incident with exactly $n + 1$ points.*

(4) Prove that in a finite affine plane of order n, if a line ℓ has exactly n points on it and A is a point not on ℓ, then there are exactly $n+1$ lines passing through A. For the proof, you should only use the axioms you stated in (2).

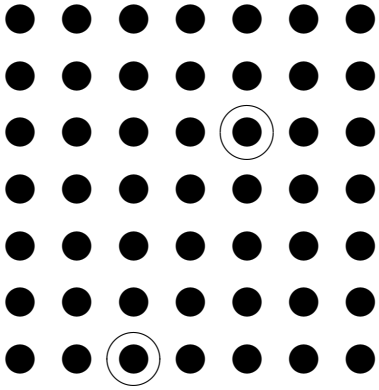
Part II. The following are miscellaneous problems related to models for finite projective planes and finite affine planes. Each problem in this section is worth 7 points.

(1) Give a model for a finite affine plane of order 2. Label each of the lines and points in your model.

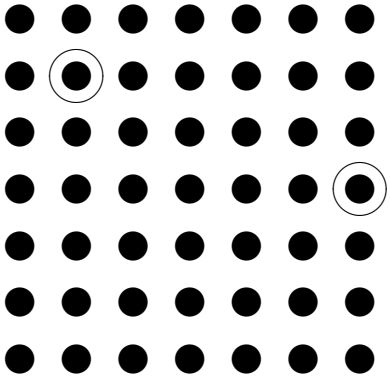
(2) Make an incidence table for the model you gave in (1). You should make use of the labelling above.

(3) Two points have been circled in the 7×7 array of points labelled "Problem 3" below. Using the model for a finite affine plane of order 7 discussed in class, finish circling the points that belong to the same line as the given circled points.

(4) Two points have been circled in the 7×7 array of points labelled "Problem 4" below. In the model of a finite projective plane of order 7 discussed in class, there is a line passing through these two points. Find the equation of this line. Put your answer in the form $y \equiv mx + b \pmod 7$ where m and b are among the numbers 0, 1, 2, 3, 4, 5, and 6.



Problem 3



Problem 4

Part III. The problems in this section all deal with an axiomatic system consisting of the axioms below. Be sure to answer the questions being asked. For example, if you are giving a model to justify your answer in problem (1) below, make sure you also state whether your answer is, "Yes" or "No." The first 4 problems in this section are worth 7 points each, and the last problem is worth 10 points.

Axiom 1. There exist 3 noncollinear points.

Axiom 2. Given any 2 distinct points, there exists exactly one line passing through the 2 points.

Axiom 3. Given any 2 distinct lines which are not parallel, there does not exist a third line parallel to both of the given lines.

Axiom 4. Given any line ℓ and any point P not on ℓ, there is at least one line parallel to ℓ that P lies on.

(1) Is the axiomatic system consistent? Justify your answer.

(2) Is the axiomatic system independent? Justify your answer.

(3) Is the axiomatic system complete? Justify your answer.

(4) Does the principle of duality hold for the axiomatic system? Justify your answer.

(5) Prove that there exist 4 points no 3 of which are collinear.