MATH 532/736I, LECTURE 7

1. **Homework:** Problem Sheet on Vector Notation

2. **Theorem 1.** Let A and B be distinct points. Then C is on $\overleftrightarrow{AB}$ if and only if there is a real number t such that $C = (1-t)A + tB$.

   Basic Ideas of Proof:

   - $\overrightarrow{AC} = t\,\overrightarrow{AB}$
   - $C - A = t\,(B - A)$

3. **Comment:** In Theorem 1,
$$\frac{t}{1-t} = \pm\frac{\text{length of } \overline{AC}}{\text{length of } \overline{CB}},$$
   where a plus sign occurs on the right if and only if C is between A and B and one denominator is 0 if and only if the other denominator is 0.

   Basic Idea of Proof: Consider 3 cases depending on the position of C relative to A and B.

4. **Theorem 2.** If A, B and C are collinear, then there exist real numbers x, y, and z not all 0 such that
$$x + y + z = 0 \quad \text{and} \quad xA + yB + zC = \overrightarrow{0}.$$

   Basic Ideas of Proof:

   - If $A = B$, take $x = 1$, $y = -1$, and $z = 0$.
   - Otherwise, use Theorem 1 and take $x = 1 - t$, $y = t$, and $z = -1$.

5. **Theorem 3.** If A, B and C are points and there exist real numbers x, y, and z not all 0 such that
$$x + y + z = 0 \quad \text{and} \quad xA + yB + zC = \overrightarrow{0},$$
   then A, B and C are collinear.

   Basic Ideas of Proof:

   - Relabel so $x \neq 0$.
   - Deduce $A = (-y/x)B + (-z/x)C$ and $1 = -y/x - z/x$.
   - Take $t = -z/x$ so that $1 - t = -y/x$.
   - Use Theorem 1.

6. **Theorem 4.** If A, B and C are not collinear and there exist real numbers x, y, and z such that
$$x + y + z = 0 \quad \text{and} \quad xA + yB + zC = \overrightarrow{0},$$
   then $x = y = z = 0$.

   Basic Idea of Proof: This is a rewording of Theorem 3.