MATH 532/736I, LECTURE 6
Wrapping Things Up

1. Recall the axioms of finite projective and affine planes both of order n.

   Axiom P1: There exist at least 4 points no 3 of which are collinear.

   Axiom P2: There exists at least 1 line with exactly $n+1$ (distinct) points on it.

   Axiom P3: Given 2 distinct points, there is exactly 1 line that they both lie on.

   Axiom P4: Given 2 distinct lines, there is at least 1 point on both of them.

   Axiom A1: There exist at least 4 points no 3 of which are collinear.

   Axiom A2: There exists at least 1 line with exactly n (distinct) points on it.

   Axiom A3: Given 2 distinct points, there is exactly 1 line that they both lie on.

   Axiom A4: Given any line ℓ and any point P not on ℓ, there is exactly 1 line through P that does not intersect ℓ.

2. Explain why the axioms for an affine plane hold for a $p \times p$ array of lines and points that we constructed. Here, p is a prime, points are of the form (a, b) where a and b are in the set, say, $S = \{0, 1, \ldots, p-1\}$, and lines are of one of two forms, $x \equiv u \pmod{p}$ or $y \equiv mx + k \pmod{p}$ where u, m and k are all in S.

3. Explain how to add $n+1$ points and 1 line to an arbitrary finite affine plane of order n to create a finite projective plane of order n. This is done by adding new points at "infinity" and adding a line that passes through all of the new points. Each new point is associated with lines of the form $x \equiv u \pmod{p}$ or lines with the same slope. The new points are used to make the parallel lines in the affine plane intersect (at a new point at infinity). Justify that the axioms for a projective plane of order n hold when the new points and line are added.

4. Explain how to create an affine plane of order n from an arbitrary projective plane of order n by taking away any line and all the points on that line. Justify, in this case, that the axioms for an affine plane will hold.