MATH 532/736I, LECTURE 4

1. Finish Previous Notes
2. Homework: Assign further problems from Homework 1.
   Quiz: 01/27/09, Tuesday
3. **Finite Projective Planes** (Given a positive integer n called the order.)

   Axiom P1: There exist at least 4 points no 3 of which are collinear.

   Axiom P2: There exists at least 1 line with exactly $n + 1$ (distinct) points on it.

   Axiom P3: Given 2 distinct points, there is exactly 1 line that they both lie on.

   Axiom P4: Given 2 distinct lines, there is at least 1 point on both of them.

4. Prove the duals of the above axioms in the following order.

   Dual of Axiom P4: Given 2 distinct points, there exists at least 1 line passing through both of them.

   Dual of Axiom P3: Given 2 distinct lines, there exists exactly 1 point on both of them.

   Dual of Axiom P1: There exist at least 4 distinct lines, no 3 of which are concurrent.

   Dual of Axiom P2: There exists at least 1 point with exactly $n + 1$ lines passing through it.

   Comments: Give handout of the proof of the dual of Axiom P1 before proving it. Before the proof of the dual of Axiom P2 mention that the following will be shown:

   Lemma. If ℓ is a line with exactly $n + 1$ points on it (in a finite projective plane of order n) and A is a point not on ℓ, then there exist exactly $n + 1$ lines passing through A.

5. **Main Steps for Lemma:**
   - Let $P_1, \ldots, P_{n+1}$ be the points on ℓ.
   - Let ℓ_j be the line through A and P_j (use Axiom P3).
   - No ℓ_j is ℓ (A is not on ℓ).
   - The ℓ_j are distinct (use Axiom P3); hence, at least $n + 1$ lines go thru A.
   - Let ℓ' be a line through A.
   - Then some P_j is on ℓ' (use Axiom P4).
   - Deduce $\ell' = \ell_j$ (by Axiom P3); so no more than $n + 1$ lines go thru A.

6. **Main Steps for Dual of P2:**
   - Let ℓ be a line with exactly $n + 1$ points on it (use Axiom P1).
   - Let A be a pont not on ℓ (use Axiom P1).
   - Apply the lemma.