MATH 241: TEST 3

Name _____

Instructions and Point Values: Put your name in the space provided above. Make sure that your test has six different pages including one blank page. Work each problem below and show ALL of your work. Do NOT use a calculator.

Point Values: Problem (1) is worth 24 points (8 points each part), Problem (2) is worth 12 points, each of Problems (3), (4), (5), and (6) is worth 16 points.

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(1) Calculate the following double integrals. **SIMPLIFY** your answers.

(a) $\displaystyle\int_0^4 \int_0^y \frac{x}{y^2}\, dx\, dy$

Answer:

(1) (continued)

(b) $\displaystyle\int_0^{\pi/2}\int_0^{2}(2-\sin\theta)\,dr\,d\theta$

Answer: $\boxed{}$

(c) $\displaystyle\int_0^{\pi/2}\int_x^{\pi/2}\frac{\cos x}{\sqrt{1+\cos y}}\,dy\,dx$

Answer: $\boxed{}$

(2) Calculate cylindrical coordinates (r, θ, z) and spherical coordinates (ρ, θ, ϕ) for the point with rectangular coordinates $(x, y, z) = (1, 0, 1)$. **SIMPLIFY** your answers (your answers should NOT involve inverse trignometric functions).

(r, θ, z):

(ρ, θ, ϕ):

(3) Calculate the line integral $\int_C (3x - 4y)\, ds$ where C is the curve given by $x = 4t + 1$ and $y = 3t - 2$ with $0 \leq t \leq 1$.

Answer:

(4) Express the volume of the solid in the first octant bounded by $z = 4 - x^2 - y^2$, the plane $y = 1$, and the coordinate planes as an iterated *triple* integral (with appropriate limits of integration). You do not need to evaluate the result.

Triple Integral: $\square$

(5) Using cylindrical coordinates, calculate the value of

$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \int_{-2}^{2} \sqrt{x^2+y^2}\, dz\, dx\, dy.$$

Answer: $\square$

(6) Using spherical coordinates, calculate the value of

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{0}^{\sqrt{4-x^2-y^2}} \frac{1}{\sqrt{(x^2+y^2)(x^2+y^2+z^2)}}\, dz\, dy\, dx.$$

Answer: