Math 241: Calculus III Name _____
Test #3

Show All Work
Points: (1) 20 pts, (2) – (6) 16 pts each

(1) Calculate the following double integrals. (You may have to switch the order of integration.)

(a) $\int_0^1 \int_y^{y^2} y \, dx \, dy$

(b) $\int_0^{\pi} \int_0^2 \sin\theta \, dr \, d\theta$

(c) $\int_0^1 \int_{\sqrt{y}}^1 e^{(x^3)} \, dx \, dy$

(2) Let $\mathbf{F} = z\,\mathbf{i} + x\,\mathbf{j} + y\,\mathbf{k}$. Calculate the divergence and curl of $\mathbf{F}$.

Divergence: ☐

Curl: ☐

(3) Calculate cylindrical coordinates (r, θ, z) and spherical coordinates (ρ, ϕ, θ) for the point with rectangular coordinates $(x, y, z) = (\sqrt{3}, -1, -2)$.

(r, θ, z): ☐

(ρ, ϕ, θ): ☐

(4) Rewrite the following integral using polar coordinates and then calculate its value.

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \left(x^2+y^2\right)^{3/2}\, dy\, dx$$

(5) Write a triple integral in spherical coordinates which represents the volume of the solid above the surface $z = \sqrt{x^2+y^2}/\sqrt{3}$ and below the surface $x^2+y^2+z^2 = 4$. (Do not evaluate the integral.)

(6) Calculate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} \frac{1}{x^2+y^2}\, dz\, dy\, dx.$$