Math 241: Test 2

Name _____

Instructions and Point Values: Put your name in the space provided above. Make sure that your test has six different pages including one blank page. Work each problem below and show ALL of your work. You do not need to simplify your answers. Do NOT use a calculator.

Point Values: Problem (1) is worth 14 points, Problem (2) is worth 12 points, Problem (3) is worth 12 points, Problem (4) is worth 12 points, Problem (5) is worth 14 points, Problem (6) is worth 18 points, and Problem (7) is worth 18 points.

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(1) For both parts of this problem, $f(x, y) = x^3y^2 - y$.

(a) Find the directional derivative of $f(x, y)$ at the point $(1, -2)$ in the direction of $\langle -3, 4\rangle$.

Directional Derivative: $\boxed{}$

(b) There are infinitely many different values for the directional derivative of $f(x, y)$ at the point $(1, -2)$ (since there are infinitely many directions that can be used to compute the directional derivative). Which of these is minimal? In other words, what is the smallest value of the directional derivative of $f(x, y)$ at the point $(1, -2)$?

Minimal Directional Derivative: $\boxed{}$

(2) Calculate f_{yyyxx} where $f(x,y) = x^3y^2e^{\sqrt{\cos x}} + x^3y^3 - 15x^4$.

Answer: $\boxed{}$

(3) Using the Chain Rule, compute $\dfrac{\partial w}{\partial u}$ where

$$w = x^3y - y^2 + \sin z,$$

$$x = u^4 + v^4, \quad y = \cos(uv), \quad \text{and} \quad z = 2^v\sqrt{v+2}$$

You do not need to put your answer in terms of u and v (the variables x, y, and z can appear in your answer).

Answer: $\boxed{}$

(4) Find an equation for the tangent plane to the surface $x^4 - y^3 + z^2 + xy = 2$ at the point $(1, 1, -1)$.

Equation of Tangent Plane: $\boxed{}$

(5) Let $R = \{(x, y) : -1 \leq x \leq 1,\ -1 \leq y \leq 1\}$ and

$$f(x, y) = \begin{cases} 1 & \text{for } x \leq y \\ 5 & \text{for } x > y. \end{cases}$$

Evaluate $\displaystyle\iint_R f(x, y)\, d\,A$.

Answer: $\boxed{}$

(6) Find the critical points of the function $f(x, y) = x^3 + 4xy^2$ where (x, y) is restricted to points in the set $S = \{(x, y) : x^2 + y^2 \leq 9\}$. Also, determine the maximum and minimum values of $f(x, y)$ in S as well as all points (x, y) where these extreme values occur.

Critical Points: $\boxed{}$

The Maximum Value is $\boxed{}$ and it occurs at the point(s) $\boxed{}$.

The Minimum Value is $\boxed{}$ and it occurs at the point(s) $\boxed{}$.

(7) Let

$$f(x, y) = xy^3 - 2x^3 + 2x^2 - 3xy,$$

and consider the points

$$P = (1, -1), \quad Q = (-1, 1), \quad \text{and} \quad R = (0, 0).$$

Exactly two of these points are critical points for $f(x, y)$ (and there are other critical points as well which you do not need to find). Determine which two of these three points are critical points and indicate (with justification) whether each determines a local maximum value of $f(x, y)$, a local minimum value of $f(x, y)$, or a saddle point of $f(x, y)$.

1) First Critical Point: $\boxed{}$ (Use a point listed above.)

Indicate Local Max, Local Min, or Saddle Point: $\boxed{}$

2) Second Critical Point: $\boxed{}$ (Use a point listed above.)

Indicate Local Max, Local Min, or Saddle Point: $\boxed{}$