Math 241: Calculus III Name _____
Test #2

Show All Work
Points: (1) 6 pts each part, (2) − (5) 13 pts each, (6) 18 pts

(1) (a) Calculate $\dfrac{\partial f}{\partial y}$ where $f(x, y) = x^2 \sin(xy)$.

(b) Calculate $\nabla f(2, 1)$ where $f(x, y) = x^2 + y^3$.

(c) Calculate f_{xxxxyy} if $f(x, y) = x^3 y^2 \sqrt{y}\, \sin(y) e^y \ln y$.

(d) Integrate $\displaystyle\int_0^1 \int_0^3 xy^2\, dx\, dy$

(e) Find the directional derivative of $f(x, y) = x^2 y + x + 2$ at the point $P = (1, 1)$ in the direction of $\mathbf{v} = -i + j$.

(2) Find the equation of the tangent plane to the surface $z^2 = x^3 + y^2$ at the point $(2, 1, -3)$.

(3) Calculate $\dfrac{\partial z}{\partial t}$ given that $z = y^2\sqrt{y}\sin(x+y)$, $x = 3u + 2v$, $y = v^3 - 12v^2 + 5v + 16$, $u = r + 2s$, $v = r^2 + 3r + 5$, $r = w^3 + \cos(w)$, and $s = 3tw$. Use any method you want. You do not need to write your answer in terms of w and t (i.e., you may have other variables in your answer). Do NOT simplify.

(4) Find every point $P = (a, b, c)$ on the surface $z = (x + y)^3 x + x^2 - x$ such that the tangent plane to the surface at P is horizontal (i.e., the tangent plane is parallel to the $xy-$plane).

(5) Using the second derivative test for functions of two variables, find all points (a, b, c) where the graph of $f(x, y) = x^2 + 2xy + 2y^2 + 2x + 1$ has a local maximum or a local minimum. For each such point, indicate which (a local maximum or a local minimum) occurs.

(6) Find the maximum and minimum values for the function $f(x, y) = xy^2 + 3y^2 + 5x - 5$ in the disk $x^2 + y^2 \leq 4$. Be sure you justify your answers.

Maximum: $\boxed{}$

Minimum: $\boxed{}$