MATH 241: TEST 1

Name _____

Instructions: Put your name in the space provided above. Check that your copy of this test contains 8 different pages. Work each problem and show ALL of your work. Unless stated otherwise, you do not need to simplify your answers. Do NOT use a calculator.

Point Values: Problem (1) is worth 30 points (6 points each part), Problem (2) is worth 18 points (6 points each part), Problem (3) is worth 12 points, Problem (4) is worth 16 points (8 points each part), and Problem (5) is worth 18 points (6 points each part).

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(1) Throughout this problem, $P = (6, 4, 0)$, $Q = (4, 1, -1)$, and $R = (7, 2, -3)$.

(a) Calculate the vector $\overrightarrow{QP}$.

$\overrightarrow{QP} =$ [ ]

(b) Calculate the magnitude of the vector $\overrightarrow{QP}$.

$|\overrightarrow{QP}| =$ [ ]

(c) Calculate $\angle PQR$ and simplify your answer (it should not involve any inverse trigonometric functions).

$\angle PQR =$ [ ]

(1) (continued)

(d) Calculate the area of ΔPQR.

Area: $\boxed{}$

(e) Determine a point S such that P, Q, R, and S are the four vertices of a parallelogram. Justify your answer. (There is more than one correct answer.)

$S =$ $\boxed{}$

(2) Let $\overrightarrow{r}(t) = \langle 6t - 1, t^3, 3t^2 \rangle$ be the position vector of a moving particle at time t.

(a) Calculate the velocity of the particle at time t.

$\overrightarrow{v}(t) =$ [ ]

(b) Calculate a unit vector that is tangent to the curve (the curve given by the position vector $\overrightarrow{r}(t)$) at time $t = 0$.

Unit Vector: [ ]

(c) Determine the length of the curve from $t = 0$ to $t = 1$.

Length: [ ]

(3) Let $\mathcal{P}$ be the plane $x + y - z = 2$. Find the equation of a plane perpendicular to $\mathcal{P}$ and passing through the points $(1, 4, -3)$ and $(1, 5, -2)$.

Equation:

(4) (a) Explain why the lines given below, ℓ_1 (to the left) and ℓ_2 (to the right), are skew (i.e., explain why they do not intersect and why they are not parallel). Justify your explanations.

$$\ell_1 : \begin{cases} x &= 2 + t \\ y &= 0 \\ z &= -1 + t \end{cases} \qquad \ell_2 : \begin{cases} x &= 3 \\ y &= 2t \\ z &= 1 + t \end{cases}$$

(b) Calculate the distance between the two lines given in part (a) (i.e., find the minimum distance from a point on ℓ_1 to a point on ℓ_2).

Distance: $\boxed{}$

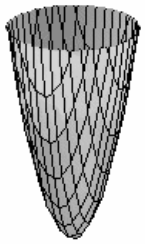
(5) The graphs for the equations below are similar to the graphs on the next page. The orientation and the scaling may be different. For each equation, indicate which graph on the next page best matches it. For example, if the equation is for a hyperbolic paraboloid, then the graph you choose should be a hyperbolic paraboloid. Indicate your choice by putting the corresponding letter from the next page after the equation below. Next, read the question on the next page corresponding to the graph you choose. Then go back to the equation below and answer the question for the graph of that equation. Do **NOT** answer the question for the graph on the next page (since it may be oriented differently than the graph of the equation below).

(i) $x^2 - y^2 - 4z^2 = -1$

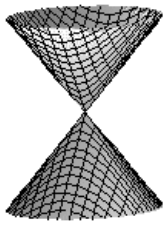
(ii) $x^2 - y^2 - 4z^2 = 0$

(iii) $x - y^2 - 4z^2 = 0$

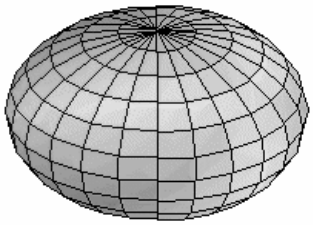
(a) This is a graph of an elliptic paraboloid. Where is the vertex of the paraboloid in the graph on the previous page? Answer with a point (give all 3 coordinates).



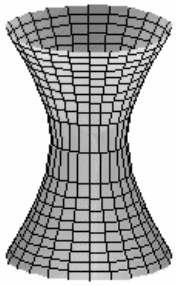
(b) This is an elliptic cone. What is the intersection of the graph for the corresponding equation on the previous page with the plane $y = 1999$? Answer either lines, an ellipse, a square, a parabola, or a hyperbola.



(c) This is an ellipsoid. Indicate where the ellipsoid intersects the y-axis for the corresponding graph on the previous page. Your answer should be two points. (Give all 3 coordinates for each point.)



(d) This is a graph of a hyperboloid of 1 sheet. There are two points on the corresponding graph on the previous page that are closer (a shorter distance) to the x-axis than the other points on the graph. Tell me one of these points (give all 3 coordinates).



(e) This is a graph of a hyperboloid of 2 sheets. There is a hyperboloid of 2 sheets that intersects the y-axis (at its vertices) and which has the same size and shape as the hyperboloid of 2 sheets given on the previous page. Write down the equation for this new hyperboloid of 2 sheets.