Math 241: Calculus III Name _____
Test #1

Show All Work
Points: (1) 3 pts each part, (2) – (7) 12 pts each, (8) 8 pts each part, Extra Credit 6 points

(1) Let $\mathbf{a} = \langle 1, 1, 2 \rangle$ and $\mathbf{b} = \langle 1, 0, 1 \rangle$. Calculate each of the following:

(a) $2\mathbf{a} - 3\mathbf{b}$

(b) $\mathbf{a} \cdot \mathbf{b}$

(c) $\text{comp}_{\mathbf{b}}\mathbf{a}$

(d) the angle θ between $\mathbf{a}$ and $\mathbf{b}$ expressed in radians with $0 \leq \theta \leq \pi$ (do not leave your answer in terms of an inverse trigonometric function – simplify)

(2) Find the volume of the pyramid with vertices $P = (0, 1, 1)$, $Q = (2, 2, 3)$, $R = (0, 4, 3)$, and $S = (0, 1, 3)$.

(3) The line given by the paremetric equations $x = 1 + t$, $y = 2$, and $z = 2 + t$ is on a plane $\mathcal{P}$. The plane $y + z = 4$ is perpendicular to plane $\mathcal{P}$. Find the equation for plane $\mathcal{P}$.

(4) The 2 planes $x - y - z = 1$ and $x + y = 2$ intersect in a line. Find a vector going in the same direction as the line.

(5) If the velocity vector at time t of a particle is $\mathbf{v}(t) = (\sin t)\mathbf{i} + (\cos t)\mathbf{j} + \mathbf{k}$ and its initial position is at the origin (that is $\mathbf{r}(0) = (0, 0, 0)$), find its position at time $t = \pi$.

(6) Describe the level curves for the function $f(x, y) = \ln(x - y)$. Be sure to include work justifying your answer for this problem. Your answer should be one of the following: lines, circles, circles together with a point, parabolas, hyperbolas, or a parallelepiped.

(7) Calculate $\lim_{(x,y)\to(0,0)} \dfrac{x^3 - 2y^3}{2x^2 + y^2}$. The work will be worth more than the answer for this problem.

(8) The graphs for the equations below are similar to 2 graphs on the last 2 pages of this test. The orientation and the scaling may be different. For each equation, indicate which graph on the last 2 pages best matches it. For example, if the equation is for a hyperbolic paraboloid, then the graph you choose should be a hyperbolic paraboloid. Indicate your choice by putting the corresponding letter from the last 2 pages in the box under the equation below. Next, read the question on the last 2 pages corresponding to the graph you chose. Then go back to the equation below and put the answer to the question for the graph of that equation in the box. Do **NOT** answer the question for the graph on the last 2 pages (since it may be oriented differently than the graph of the equation below).

(a) $x^2 - 2y^2 = 3z^2$

ANSWER: [ ]

(b) $x^2 - 2y^2 = 3z^2 - 1$

ANSWER: [ ]

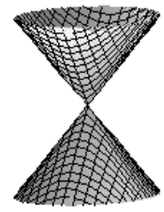
Extra Credit:

In class I said something like the following: if $f(x, y)$ and $g(x, y)$ are polynomials in 2 variables and the degree of $f(x, y)$ is greater than the degree of $g(x, y)$, then

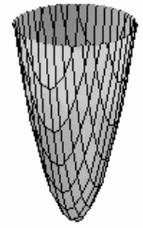
$$\lim_{(x,y)\to(0,0)} \frac{f(x, y)}{g(x, y)} = 0.$$

Give a counterexample. In other words, find polynomials $f(x, y)$ and $g(x, y)$ with the degree of $f(x, y)$ greater than the degree of $g(x, y)$ and with the above limit NOT being 0. Be sure to justify your answer.

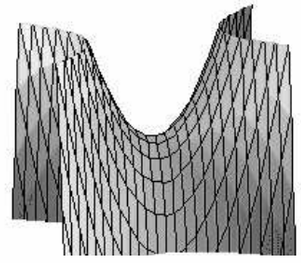
(a) This is a graph of an elliptic cone. Planes parallel to the $xy-$plane intersect this graph in ellipses (except for the plane $z = 0$ which gives only a point). The equation on page 5 corresponding to this graph produces ellipses when intersected by the right planes. What are they parallel to (one of the xy-plane, the xz-plane, or the yz-plane)?



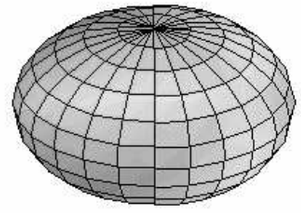
(b) This is a graph of an elliptic paraboloid. What are the coordinates of the vertex for the corresponding equation on page 5?



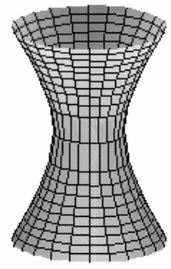
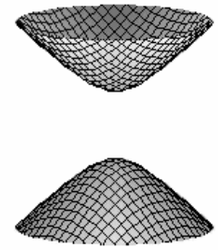
(c) This is a hperbolic paraboloid. There is a plane $z = k$ which intersects the graph for the corresponding equation on page 5 in 2 lines. What is the value of k and what are the equations of the 2 lines?



(d) This is an ellipsoid. What is the center of the ellipsoid for the graph of the corresponding equation on page 5.



(e) This is a graph of a hyperboloid of 1 sheet. There is a number $k > 0$ such that the plane $z = k$ intersects the graph for the corresponding equation on page 5 in 2 lines. What is the value of k and what are the equations of the 2 lines?



(f) This is a graph of a hyperboloid of 2 sheets. What is the intersection of the corresponding graph on page 5 with planes parallel to the yz-plane? Answer either lines, ellipses, parabolas, or hyperbolas.