Answers to Test 3, Spring 2001

1. (a) 1
   (b) $\pi^3/6$
   (c) $\dfrac{1}{6}\sin\left(\dfrac{\pi^3}{8}\right)$

2. $(r,\theta,z) = (\sqrt{2}, 7\pi/6, -\sqrt{2})$

   $(\rho,\theta,\phi) = (2, 7\pi/6, 3\pi/4)$

   Comment: To calculate ϕ, be sure to use $z = \rho\cos\phi$. Using $r = \rho\sin\phi$ causes problems. The point here is that $\cos\phi$ takes on different values for different ϕ with $0 \le \phi \le \pi$ and $\sin\phi$ does not.

3. 3

4. $\sqrt{2/5} = \sqrt{10}/5$

5. (a) $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{2-y^2}} dz\,dy\,dx = \int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{\sqrt{2-y^2}} dz\,dx\,dy$
   (b) $\displaystyle\int_0^{\pi/2} \int_0^1 \int_0^{\sqrt{2-r^2\sin^2\theta}} r\,dz\,dr\,d\theta$

6. $\dfrac{4^{10}\pi}{10} = \dfrac{2^{19}\pi}{5} = \dfrac{524288\pi}{5}$