Math 241: Test 2

Name ______________________________

Instructions and Point Values: Put your name in the space provided above. Make sure that your test has seven different pages including one blank page. Work each problem below and show ALL of your work. You do not need to simplify your answers. Do NOT use a calculator.

Point Values: Problems (1) through (7) are worth 10 points each, Problem (8) is worth 14 points, and Problem (9) is worth 16 points.

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(1) Let

$$f(x,y) = \frac{2x^3 - 3y^2}{x^2 + y^2}.$$

Does $\lim_{(x,y)\to(0,0)} f(x,y)$ exist? If so, what is it? If not, why not?

(2) Calculate f_{yyxx} where $f(x,y) = x^3y^2 - y^3x^2 + x\cos(\sqrt{x}) + y\cos x^3$.

Answer: $\boxed{}$

(3) Let $R = \{(x,y) : 1 \le x \le 4, 1 \le y \le 4\}$ and

$$f(x,y) = \begin{cases} 1 & \text{for } 1 \le x < 3 \text{ and } 1 \le y \le 3 \\ -1 & \text{for } 3 \le x \le 4 \text{ and } 1 \le y \le 4 \\ 2 & \text{for } 1 \le x < 3 \text{ and } 3 < y \le 4. \end{cases}$$

Evaluate $\displaystyle\iint_R f(x,y)\, dA$.

Answer: $\boxed{}$

(4) Calculate $\int_0^1 \int_0^2 \left(3x^2 - 2xy\right) dx\, dy$.

Answer: $\boxed{}$

(5) Find an equation for the tangent plane to the surface $xy^2z^3 = 3$ at the point $(-3, 1, -1)$.

Equation of Tangent Plane: $\boxed{}$

(6) A drop of water is placed gently onto the surface of $z = 3y^2 - 4x^2$ at the point $(1, 1, -1)$. In what direction does the drop begin to move (assume it goes downward in the direction of the steepest incline)? Express your answer as a unit vector $\langle a, b \rangle$ (so the raindrop will go in this direction along the surface).

Unit Vector: $\boxed{}$

(7) Using the Chain Rule, compute $\dfrac{\partial w}{\partial t}$ where

$$w = z^2y + \cos(xy) + z^3,$$

$$x = s - t, \qquad y = s^2 + t, \qquad \text{and} \qquad z = s^2 + 9.$$

You do not need to put your answer in terms of s and t (the variables x, y, and z can appear in your answer).

Answer: $\boxed{}$

(8) Determine the global maximum and minimum values for the function

$$f(x, y) = x^3 + 4x^2 + 4y^2 - 3x$$

on the set

$$S = \{(x, y) : x^2 + y^2 \leq 4\}.$$

Also, indicate *all* points (x, y) in S where these values occur. (Note that the boundary of S is the circle of radius 2 centered at the origin.)

The Global Maximum Value is $\boxed{}$ and it occurs at the point(s) $\boxed{}$.

The Global Minimum Value is $\boxed{}$ and it occurs at the point(s) $\boxed{}$.

(9) Let

$$f(x,y) = x^2y - 2xy^2 - 2y^2 - y.$$

The following partial derivatives can be computed for this function (you do not need to compute them yourself):

$$f_x = 2y(x-y), \qquad f_y = (x+1)(x-4y-1),$$

$$f_{xx} = 2y, \qquad f_{xy} = 2x - 4y, \qquad \text{and} \qquad f_{yy} = -4x - 4.$$

There are four critical points for $f(x,y)$. Determine them and indicate (with justification) whether each determines a local maximum value of $f(x,y)$, a local minimum value of $f(x,y)$, or a saddle point of $f(x,y)$.

1) First Critical Point: ☐

Indicate Local Max, Local Min, or Saddle Point: ☐

2) Second Critical Point: ☐

Indicate Local Max, Local Min, or Saddle Point: ☐

3) Third Critical Point: ☐

Indicate Local Max, Local Min, or Saddle Point: ☐

4) Fourth Critical Point: ☐

Indicate Local Max, Local Min, or Saddle Point: ☐