Name: _____

Math 241: Test 1, Fall 2020

Instructions and Point Values: Make sure you have 6 different pages with this test, including one blank page (so 5 test pages). Put your name at the top of this page. There are 5 problems. Show ALL of your work. Your WORK will be graded. Put your answers in the boxes below. Unless stated otherwise, you do not need to simplify your answers. Do NOT use a calculator. The point value for each problem appears to the left of each problem.

35 pts (1) Let $A = (2, 4, -1)$, $B = (-2, 8, 1)$ and $C = (6, -1, 2)$. Fill in the boxes below by doing an appropriate calculation. As noted above, your work will be graded.

(a) The distance from A to the y-axis is $\boxed{}$. Simplify.

(b) $\overrightarrow{AB} = \boxed{}$ and $\overrightarrow{AC} = \boxed{}$. Simplify.

(c) The distance from A to B is $\boxed{}$. Simplify.

(d) A vector which has the opposite direction as $\overrightarrow{AB}$ and which has length 3 is $\boxed{}$. Simplify so the answer only involves integers (no fractions).

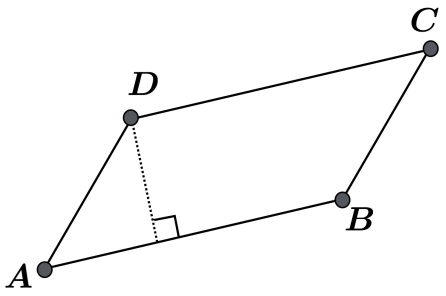
(e) The measure of $\angle BAC$ is $\boxed{}$. Simplify. Do not use an inverse trig function.

16 pts (2) The four vertices of a parallelogram are given by

$$A = (0, 0, 0), \quad B = (2, 1, 2),$$

$$C = (4, 1, 6) \quad \text{and} \quad D = (2, 0, 4).$$

The sides are $\overline{AB}$, $\overline{BC}$, $\overline{CD}$ and $\overline{AD}$.



(not drawn to scale)

(a) Using the vectors $\overrightarrow{AB}$ and $\overrightarrow{AD}$, compute the area of the parallelogram.

Area: [ ] (Simplify)

(b) What is the height of the parallelogram using $\overline{AB}$ as the base?

Height: [ ] (Simplify)

15 pts (3) The plane $\mathcal{P}_1$ is given by the equation

$$6x - 3y + 2z = 2,$$

and the plane $\mathcal{P}_2$ is given by the equation

$$6x - 3y + 2z = 6.$$

The planes $\mathcal{P}_1$ and $\mathcal{P}_2$ are parallel (you do not need to justify this). Using the length of a projection vector, calculate the distance between the two planes. For full credit, your calculation for the answer must involve calculating the length of a projection.

Distance between planes: $\boxed{}$ (Simplify)

16 pts (4) (a) Write parametric equations for the line ℓ that passes through the point $P = (1, -1, 1)$ and is parallel to the line with parametric equations $x = 2 - t$, $y = 3t$ and $z = 1 + t$.

Parametric Equations of Line ℓ:

(b) At what point does your answer to part (a) intersect the plane $3x + y + z = 0$?

Point of Intersection: (Simplify)

18 pts (5) The graphs for the equations below are similar to the graphs given in the "Graph Section" below. The orientation and scaling may be different. For each equation, indicate which graph in the Graph Section best matches it. For example, if the equation is for a hyperbolic paraboloid, then the graph you choose should be a hyperbolic paraboloid. Indicate your choice by writing down and circling the corresponding letter from the Graph Section next to the problem - next to (i), (ii) or (iii) below. Next, read the question in the Graph Section corresponding to the graph you choose. Then answer the question for the graph of the equation in the problem below. Do **NOT** answer the question for the graph in the Graph Section (since it may be oriented differently than the graph of the equation below).

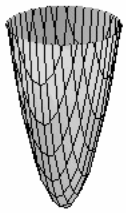
(i) $3x^2 - y^2 - 4z^2 + 4 = 0$

(ii) $3x^2 - y^2 - 4z^2 = 0$

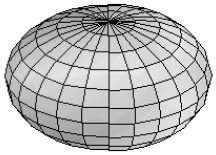
(iii) $3x^2 - y^2 - 4z + 4 = 0$

Graph Section

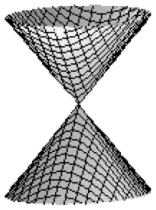
(a) To the right is a graph of an elliptic paraboloid. What point is the vertex in the graph for the problem? Give all 3 of the coordinates.



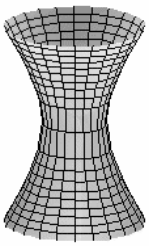
(b) This is a graph of an ellipsoid. What are the 2 points where the graph for the problem intersects the y-axis? Give all 3 coordinates for each point.



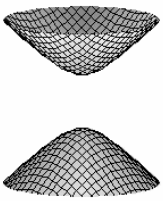
(c) This is an elliptic cone. The xy-plane intersects the graph for the problem in two lines. Write down parametric equations for these two lines.



(d) This is a hyperboloid of 1 sheet. What is the intersection of the plane $z = 1$ with the graph in the problem? Answer a point, two lines, an ellipse, a parabola, or a hyperbola.



(e) This is a hyperboloid of 2 sheets. What is the intersection of the plane $x = 2020$ with the graph in the problem? Answer a point, two lines, an ellipse, a parabola, or a hyperbola.



(f) This is a hyperbolic paraboloid. What is the intersection of the plane $y = 2020$ with the graph in the problem? Answer a point, two lines, an ellipse, a parabola, or a hyperbola.