Answers to Test 1, Fall 2001

1. (a) $\langle 2, -2, 2\rangle$
   (b) $\langle 1, -8, 3\rangle$
   (c) $\langle 0, 0, 0\rangle$

2. (a) $9/2$
   (b) 8

3. (a) $z_1(x_1+y_1)+z_2(x_2+y_2)+z_3(x_3+y_3)$
   (b) $z_1(x_1+y_1)+z_2(x_2+y_2)+z_3(x_3+y_3)$ (possibly written differently)
   (c) From (a) and (b), it follows that $\vec{w}\cdot(\vec{u}+\vec{v}) = \vec{w}\cdot\vec{u}+\vec{w}\cdot\vec{v}$. Take $\vec{w}=\vec{u}+\vec{v}$ to get
   $$(\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) = (\vec{u}+\vec{v})\cdot\vec{u}+(\vec{u}+\vec{v})\cdot\vec{v}.$$
   Taking $\vec{w}=\vec{u}$ and $\vec{w}=\vec{v}$, we obtain
   $$(\vec{u}+\vec{v})\cdot\vec{u} = \vec{u}\cdot(\vec{u}+\vec{v}) = \vec{u}\cdot\vec{u}+\vec{u}\cdot\vec{v} \quad \text{and} \quad (\vec{u}+\vec{v})\cdot\vec{v} = \vec{v}\cdot(\vec{u}+\vec{v}) = \vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{v}.$$
   Combing the equations above, we deduce
   $$(\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) = \big(\vec{u}\cdot\vec{u}+\vec{u}\cdot\vec{v}\big)+\big(\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{v}\big) = \vec{u}\cdot\vec{u}+2\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{v}.$$
   So (a) and (b) can be used to obtain the equation in the problem.
   (d) Use (c) to deduce that
   $$\begin{aligned} 36 &= |\vec{u}+\vec{v}|^2 = (\vec{u}+\vec{v})\cdot(\vec{u}+\vec{v}) \\ &= \vec{u}\cdot\vec{u}+2\vec{u}\cdot\vec{v}+\vec{v}\cdot\vec{v} = |\vec{u}|^2+2\vec{u}\cdot\vec{v}+|\vec{v}|^2 = 16+2\vec{u}\cdot\vec{v}+9. \end{aligned}$$
   It follows that
   $$\vec{u}\cdot\vec{v} = \frac{36-16-9}{2} = \frac{11}{2}.$$
   From $\vec{u}\cdot\vec{v} = |\vec{u}|\,|\vec{v}|\,\cos\theta = 12\cos\theta$, we obtain that the answer is $\boxed{11/24}$.

4. (a) $\langle t^2\cos t+2t\sin t, t^2\sin t-2t\cos t, 2\rangle$
   (b) t^2+2
   (c) 15

5. (a) $2x-y=0$ (but most wrote $4x-2y=0$ which is fine)
   (b) $(3/2, 1/2, -1/2)$

6. There are infinitely many correct answers here. One is $x+y=1$.

7. (i) (d), y-axis
   (ii) (e), $z=-1$
   (iii) (a), $(0,1,0)$