Answers to Test 1, 1999

1. (a) $\langle 2, 3, 1\rangle$
   (b) $\sqrt{14}$
   (c) $\pi/3$
   (d) $7\sqrt{3}/2$
   (e) $S = (5, -1, -4)$ or $S = (9, 5, -2)$ or $S = (3, 3, 2)$

2. (a) $\langle 6, 3t^2, 6t\rangle$
   (b) $\langle 1, 0, 0\rangle$
   (c) 7

3. $2x - y + z = -5$

4. (a) If there is a $P = (x, y, z)$ on ℓ_1 and ℓ_2, then there is some t and some s such that
$$(x, y, z) = (2 + t, 0, -1 + t) = (3, 2s, 1 + s).$$
Since $2 + t = 3$, $t = 1$. Since $0 = 2s$, $s = 0$. But then $-1 + t = 0$ and $1 + s = 1$ so that $-1 + t \neq 1 + s$. This implies that P cannot exist. In other words, ℓ_1 and ℓ_2 do not intersect. Note now that $\langle 1, 0, 1\rangle$ is a vector parallel to ℓ_1 and $\langle 0, 2, 1\rangle$ is a vector parallel to ℓ_2. Also, $\langle 1, 0, 1\rangle \neq c\langle 0, 2, 1\rangle$ for any number c (otherwise, $1 = c \times 0$, which is impossible). Hence, ℓ_1 and ℓ_2 are not parallel.
   (b) $2/3$

5. (i) (d), $(0, 0, 1/2)$ (or $(0, 0, -1/2)$)
   (ii) (b), a hyperbola
   (iii) (a), $(0, 0, 0)$