Math 241: Quiz 7 Spring, 2020 SOLUTIONS

1. Calculate an equation for the tangent plane to the surface

$$2(x-2)^2 + (y-1)^2 + (z-3)^2 = 10$$

at the point $(3, 3, 5)$.

Equation of tangent plane $\mathcal{P}$: $\boxed{x + y + z = 11}$

Solution: Let $F(x, y, z) = 2(x-2)^2 + (y-1)^2 + (z-3)^2$ (or $F(x, y, z) = 2(x-2)^2 + (y-1)^2 + (z-3)^2 - 10$). Then $\nabla F = \langle 4(x-2), 2(y-1), 2(z-3) \rangle$. Hence, $\nabla F(3, 3, 5) = \langle 4, 4, 4 \rangle = 4\langle 1, 1, 1 \rangle$. Thus, $\langle 1, 1, 1 \rangle$ is normal to the tangent plane and $(3, 3, 5)$ is a point on the plane, and we deduce that the equation for the tangent plane is $x + y + z = 11$. ■

2. Let $f(x, y) = x^2 - y^2 + 1$, and let P be the point $(0, 1)$. There are infinitely many different values for the directional derivative of $f(x, y)$ at the point P (since there are infinitely many directions that can be used to compute the directional derivative). Which of these is *minimal*? In other words, what is the least value of the directional derivative of $f(x, y)$ at the point P?

Least value of directional derivative at P: $\boxed{-2}$

Solution: Here, $\nabla f = \langle 2x, -2y \rangle$ so that $\nabla f(P) = \langle 0, -2 \rangle$ (recalling $P = (0, 1)$). The directional derivative is minimized at P when one goes in the direction of $-\nabla f(P)$. One can now compute the directional derivative in this direction. On the other hand, we also know that the minimal value of the directional derivative when we do this is $-\|\nabla f(P)\|$. Since $\|\nabla f(P)\| = \sqrt{0^2 + (-2)^2} = \sqrt{4} = 2$, the answer is -2. ■