

Math 241: Quiz 7

Show ALL Work

Name _____

Solutions

1. Determine all points (x, y) where $f(x, y) = x^2 - 4x + xy^2$ has a local minimum, a local maximum or a saddle point. There should be 3 such points. Indicate next to each point, whether the point is the location of a local minimum, a local maximum or a saddle point.

First Point: $(0, 2)$, Location of a saddle point

Second Point: $(0, -2)$, Location of a saddle point

Third Point: $(2, 0)$, Location of a local minimum

Solution: Setting the partial derivatives to 0, we obtain

$$f_x = 2x - 4 + y^2 = 0 \quad \text{AND} \quad f_y = 2xy = 0.$$

The second of these implies that $x = 0$ OR $y = 0$. If $x = 0$, then the first equation gives $-4 + y^2 = 0$, so $y = \pm 2$. If $y = 0$, then $2x - 4 = 0$, so $x = 2$. This gives us the three points $(0, 2)$, $(0, -2)$ and $(2, 0)$. For “local” extrema, we want to use

$$D(x, y) = f_{xx}f_{yy} - f_{xy}^2 = 2 \cdot 2x - (2y)^2 = 4x - 4y^2.$$

Since $D(0, \pm 2) = -16 < 0$, there are saddle points at $(0, 2)$ and $(0, -2)$. Since $D(2, 0) = 8 > 0$ and $f_{xx}(2, 0) = 2 > 0$, there is a local minimum at $(2, 0)$. ■