Math 241: Quiz 4

Show ALL Work Name **Solutions**

1. Calculate an equation for the plane that passes through the point $(2, 0, 1)$ and is parallel to the plane $2x - y + 3z = -4$.

   Equation for Plane: $\boxed{2x - y + 3z = 7}$

   Solution: The vector $\langle 2, -1, 3\rangle$ is normal to the plane $2x-y+3z = -4$ and, therefore, will be normal to the parallel plane. Hence, an equation for the parallel plane is $2x - y + 3z = d$ for some number d. Since the parallel plane passes through $(2, 0, 1)$, we see that $d = 2 \cdot 2 - 0 + 3 \cdot 1 = 7$. Thus, $2x - y + 3z = 7$ is an equation for the parallel plane. ■

2. The plane $\mathcal{P}$ consists of the points (x, y, z) satisfying $x - y + z = 0$. The point $A = (1, 2, 3)$ is not on the plane $\mathcal{P}$. Find a point B on the plane $\mathcal{P}$ such that the distance from A to B is minimal. In other words, what is the point B on the plane $\mathcal{P}$ that is nearest to the point A?

   $B = \boxed{(1/3, 8/3, 7/3)}$

   Solution 1: The line $\overleftrightarrow{AB}$ through A and B is perpendicular to the plane $\mathcal{P}$, so the normal $\overrightarrow{n} = \langle 1, -1, 1\rangle$ to $\mathcal{P}$ is parallel (in the direction of) $\overleftrightarrow{AB}$. Since $A = (1, 2, 3)$ is on $\overleftrightarrow{AB}$, the points on line $\overleftrightarrow{AB}$ satisfy the parametric equations $x = 1 + t$, $y = 2 - t$ and $z = 3 + t$. The point B is the intersection of this line with the plane $\mathcal{P}$. Since $\mathcal{P}$ is given by the equation $x - y + z = 0$, the intersection of $\overleftrightarrow{AB}$ with $\mathcal{P}$ is determined by the value of t satisfying $(1 + t) - (2 - t) + (3 + t) = 0$. This simplifies to $2 + 3t = 0$ or $t = -2/3$. Hence, the coordinates of B are given by $x = 1 + (-2/3) = 1/3$, $y = 2 - (-2/3) = 8/3$ and $z = 3 + (-2/3) = 7/3$. In other words, $B = (1/3, 8/3, 7/3)$. ■

   Solution 2: The vector $\overrightarrow{n} = \langle 1, -1, 1\rangle$ is normal to the plane $\mathcal{P}$. The point $C = (0, 0, 0)$ satisfies the equation $x - y + z = 0$ and so is on $\mathcal{P}$. Observe that $\overrightarrow{AC} = \langle -1, -2, -3\rangle$, and

   $$\overrightarrow{AB} = \text{proj}_{\overrightarrow{n}}\overrightarrow{AC} = \frac{\overrightarrow{n} \bullet \overrightarrow{AC}}{\|\overrightarrow{n}\|^2}\,\overrightarrow{n} = \frac{-2}{3}\langle 1, -1, 1\rangle = \left\langle \frac{-2}{3}, \frac{2}{3}, \frac{-2}{3} \right\rangle.$$

   We can view $\overrightarrow{AB}$ as telling us how much each coordinate of A has to change to go from A to B. Thus, $B = (1 + (-2/3), 2 + (2/3), 3 + (-2/3)) = (1/3, 8/3, 7/3)$. ■