

Math 241: Quiz 2

Show ALL Work

Name _____

Solutions

1. Circle two (and only two) of the following vectors that are perpendicular, and justify that they are perpendicular by showing appropriate work (so that I can see that you have a legitimate reason for your answer).

$$\langle 3, 1, 4 \rangle \quad \langle 2, 1, -2 \rangle \quad \langle -2, 3, -1 \rangle \quad \langle 5, 1, -4 \rangle$$

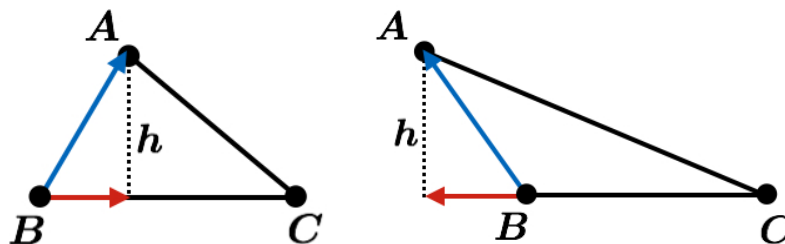
Solution: Two vectors are perpendicular if and only if their dot product is zero. Checking the various possibilities leads to

$$\langle 3, 1, 4 \rangle \cdot \langle 5, 1, -4 \rangle = 15 + 1 - 16 = 0,$$

so the vectors $\langle 3, 1, 4 \rangle$ and $\langle 5, 1, -4 \rangle$ should be circled above. ■

2. Let $A = (2, 0, 1)$, $B = (1, -2, 3)$ and $C = (4, -2, 7)$. Using a projection vector (or the length of a projection vector), find the height of $\triangle ABC$ drawn from the vertex A to base \overline{BC} . Show work and simplify your answer. (Note: I am not saying that the length of the projection vector equals the height.)

Answer: $2\sqrt{2}$



Solution: You should think in terms of one of the above pictures. Either way the work is the same. You want to compute $\|\text{proj}_{\overrightarrow{BC}} \overrightarrow{BA}\|$ and $\|\overrightarrow{BA}\|$ and use the Pythagorean Theorem to get h . Since

$$\overrightarrow{BA} = \langle 1, 2, -2 \rangle \quad \text{and} \quad \overrightarrow{BC} = \langle 3, 0, 4 \rangle,$$

we obtain

$$\|\text{proj}_{\overrightarrow{BC}} \overrightarrow{BA}\| = \frac{|\overrightarrow{BC} \cdot \overrightarrow{BA}|}{\|\overrightarrow{BC}\|} = \frac{|3 - 8|}{\sqrt{3^2 + 4^2}} = 1 \quad \text{and} \quad \|\overrightarrow{BA}\| = \sqrt{1^2 + 2^2 + (-2)^2} = 3.$$

Therefore, $h = \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$. (Note that $\overrightarrow{BC} \cdot \overrightarrow{BA} = -5 < 0$ implies that $\angle ABC > 90^\circ$ so the second picture above is more accurate than the first.) ■