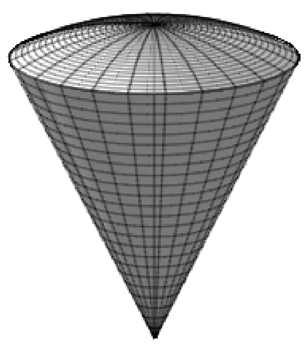
Math 241: Quiz 10 Fall, 2019 SOLUTIONS

1. Write down a triple integral *in cylindrical coordinates* that represents the volume of the solid inside the cone $z = \sqrt{4x^2 + 4y^2}$ and below the sphere $z = \sqrt{45 - x^2 - y^2}$.

Triple Integral in Cylindrical Coordinates: $\boxed{\displaystyle\int_0^{2\pi} \int_0^3 \int_{2r}^{\sqrt{45-r^2}} r\, dz\, dr\, d\theta}$



Yes, I am a picture of the solid.

Solution: The cone and sphere intersect when $\sqrt{4x^2 + 4y^2} = \sqrt{45 - x^2 - y^2}$ which after squaring and simplifying gives $x^2 + y^2 = 9$. So we are interested in points in the xy-plane which are inside the circle of radius 3 centered at the origin. In polar coordinates, we therefore want $0 \le \theta < 2\pi$ and $0 \le r \le 3$. The two equations $z = \sqrt{4x^2 + 4y^2}$ and $z = \sqrt{45 - x^2 - y^2}$ convert to $z = \sqrt{4r^2} = 2r$ and $z = \sqrt{45 - r^2}$. This gives the triple integral in cylindrical coordinates as shown. ■

2. Fill in the six boxes below to correctly complete interchanging the order of integration.

$$\int_0^4 \int_0^{(12-3x)/4} \int_0^{(12-3x-4y)/2} f(x, y, z)\, dz\, dy\, dx$$

$$= \int_{\boxed{0}}^{\boxed{6}} \int_{\boxed{0}}^{\boxed{(6-z)/2}} \int_{\boxed{0}}^{\boxed{(12-4y-2z)/3}} f(x, y, z)\, dx\, dy\, dz$$

Solution: The picture to the right can be an optical illusion, so look at it correctly. The x-axis is numbered from 0 to 4 and is coming outward from the page. The plane on the right is given by $z = (12 - 3x - 4y)/2$ or equivalently by $3x + 4y + 2z = 12$. This intersects the yz-plane (the plane $x = 0$) at the line $4y + 2z = 12$, simplifying to $2y + z = 6$. Using this information as a guide produces the answers above. ■