MATH 174, ANSWERS TO TEST 2

1. $\sum_{k=1}^{\boxed{100}} \boxed{(-1)^{k+1}2^k}$

2. $a = -3$, $b = 101$, and $c = -2$

3. (a) $\{1, 2, 3, 5\}$
   (b) $\{1, 4, 5\}$
   (c) $\{3\}$
   (d) $\{(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5), (5, 2), (5, 3), (5, 5)\}$
   (e) $\{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

4. $A^c \cup (B^c \cup C^c)$

5. True. (Appropriate Venn Diagrams would be a justification.)

6. Yes. Since $A \cup B \cup C = S$ and since the sets A, B, and C are mutually disjoint, $\{A, B, C\}$ is a partition of S.

7. 900

8. 59

9. 24

10. 120

11. 24

12. 24

13. 60

14. 56

15. $3^4\binom{7}{4}$

16. -128

17. $1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1$

18. Let $P(n)$ be the statement that the sum of the first n odd integers is n^2. We prove that $P(n)$ is true for every integer ≥ 2 by using $\boxed{\text{induction}}$. The sum of the first two odd numbers is $1 + 3 = 4$ which shows that $\boxed{P(2)}$ is true. Next, we suppose that k is an integer ≥ 2 such that $\boxed{P(k)}$ is true. This is called the $\boxed{\text{induction}}$ hypothesis. Since the k^{th} odd number is $2k - 1$, we obtain from $P(k)$ that

$$1 + 3 + 5 + \cdots + (2k - 1) = \boxed{k^2}. \qquad (*)$$

The odd number after $2k - 1$ is $2k + 1$. Therefore, we want to show that

$$1 + 3 + 5 + \cdots + (2k - 1) + (2k + 1) = \boxed{(k + 1)^2}.$$

Observe that $k^2 + (2k + 1) = (k + 1)^2$. From $(*)$, we deduce that

$$\begin{aligned} 1 + 3 + 5 + \cdots + (2k - 1) + (2k + 1) &= \big(1 + 3 + 5 + \cdots + (2k - 1)\big) + (2k + 1) \\ &= \boxed{k^2 + (2k + 1)} \text{ (use } (*)\text{)} \\ &= (k + 1)^2. \end{aligned}$$

We have shown that if $P(k)$ is true, then $\boxed{P(k + 1)}$ is true. This completes the induction argument. Therefore, $P(n)$ is true for every integer ≥ 2. ■