Math 174: Test 2

Name ________________________________

Instructions: Put your name in the space provided above. Make sure that your test has seven different pages including one blank page. Work each problem below and show <u>ALL</u> of your work. Put all answers in the spaces provided. Do <u>NOT</u> use a calculator.

Point Values: The point values are indicated to the left of each problem.

(5) 1. Complete the summation notation below by filling in the blanks appropriately. The blank above the summation should be a number > 50.

$$2 - 2^2 + 2^3 - 2^4 + \cdots + 2^{99} - 2^{100} = \sum_{k=1}^{\boxed{}} \boxed{}$$

(5) 2. Determine integers a, b, and c satisfying

$$2 - 2^2 + 2^3 - 2^4 + \cdots + 2^{99} - 2^{100} = \frac{1}{a}\left(2^b + c\right).$$

$a = \boxed{}$

$b = \boxed{}$

$c = \boxed{}$

(10) 3. The sets $A = \{1, 2, 3\}$, $B = \{1, 3, 4, 5\}$, and $C = \{2, 3, 5\}$ are subsets of the universal set $U = \{1, 2, 3, 4, 5\}$. Calculate the following.

(a) $A \cup (B \cap C)$

Answer: $\boxed{}$

(b) $B - (A \cap C)$

Answer: $\boxed{}$

(c) $B - (A \cap C)^c$

Answer: $\boxed{}$

(d) $C \times C$

Answer: $\boxed{}$

(e) $\mathcal{P}(A)$ (the powerset of A)

Answer: $\boxed{}$

(5) 4. Which one of

$$A^c \cup \left(B^c \cup C^c\right), \quad A^c \cup \left(B^c \cap C^c\right), \quad A^c \cap \left(B^c \cup C^c\right), \quad \text{and } A^c \cap \left(B^c \cap C^c\right)$$

is the same set as

$$\left(A \cap \left(B \cap C\right)\right)^c ?$$

Answer: $\boxed{}$

(6) 5. Determine whether the following is true or false for all sets A, B, and C and justify your answer. If there exist sets A, B, and C for which it is false, indicate that it is "false". Otherwise indicate that it is "true".

$$\big(A - B\big) \cap C = \big(A \cap C\big) - B$$

True or False: $\boxed{}$

Justification:

(6) 6. Do the sets $A = \{0\}$, $B = \{1, 3, 5, 7, 9\}$, and $C = \{2, 4, 6, 8, 10\}$ partition

$$S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}\,?$$

(In other words, is $\{A, B, C\}$ a partition of S?) Explain your answer.

Yes or No: $\boxed{}$

Explanation:

(5) 7. How many integers are there with exactly three digits?

(5) 8. If the largest of 75 consecutive integers is 133, what is the smallest?

(5) 9. In how many different ways can the letters in FOUR be arranged in a row?

Answer:

(5) 10. In how many different ways can the letters in ELEVEN be arranged in a row? (Each arrangement will consist of six letters, three of which are the same.)

Answer:

(5) 11. In how many different ways can the letters in ELEVEN be arranged in a row if all three E's are to be together?

Answer:

(5) 12. In how many different ways can the letters in ELEVEN be arranged in a row if no two E's are to be next to each other?

Answer:

(5) 13. There are 6 women and 4 men wanting to form a committee using 2 of the women and 3 of the men. How many different committees are possible? Simplify your answer.

Answer:

(5) 14. There are 6 women and 4 men wanting to form a committee using 2 of the women and 3 of the men. But 2 of the women, Jill and Carrie, are not both able to serve on the committee. How many different committees are possible? Simplify your answer.

Answer:

(5) 15. If $(x-3y)^7$ is expanded using the binomial theorem, what will be the coefficient of x^3y^4? Your answer does *not* need to be simplified (it may involve a power of 3 and a binomial coefficient).

Answer: $\boxed{}$

(5) 16. If $(x-3y)^7$ is expanded using the binomial theorem, the result contains 8 terms. More precisely, the expansion will be of the form

$$a_7x^7 + a_6x^6y + a_5x^5y^2 + a_4x^4y^3 + a_3x^3y^4 + a_2x^2y^5 + a_1xy^6 + a_0y^7.$$

What is the value of

$$a_7 + a_6 + a_5 + a_4 + a_3 + a_2 + a_1 + a_0\,?$$

Answer: $\boxed{}$

(5) 17. The 7^{th} row of Pascal's triangle is

$$1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1.$$

What is the 8^{th} row of Pascal's triangle?

Answer: $\boxed{}$

(8) 18. Complete the following proof that for each integer $n \geq 2$, the sum of the first n odd integers is n^2.

Let $P(n)$ be the statement that the sum of the first n odd integers is n^2. We prove that $P(n)$ is true for every integer ≥ 2 by using $\boxed{}$.

The sum of the first two odd numbers is $1+2=3$ which shows that $\boxed{}$ is true. Next, we suppose that k is an integer ≥ 2 such that $\boxed{}$ is true. This is called the $\boxed{}$ hypothesis. Since the k^{th} odd number is $2k-1$, we obtain from $P(k)$ that

$$(*) \qquad 1+3+5+\cdots+(2k-1) = \boxed{}.$$

The odd number after $2k-1$ is $2k+1$. Therefore, we want to show that

$$1+3+5+\cdots+(2k-1)+(2k+1) = \boxed{}.$$

Observe that $k^2+(2k+1)=(k+1)^2$. From $(*)$, we deduce that

$$\begin{aligned}1+3+5+\cdots+(2k-1)+(2k+1) &= \big(1+3+5+\cdots+(2k-1)\big)+(2k+1)\\ &= \boxed{} \quad \text{(use } (*)\text{)}\\ &= (k+1)^2.\end{aligned}$$

We have shown that if $P(k)$ is true, then $\boxed{}$ is true. This completes the induction argument. Therefore, $P(n)$ is true for every integer ≥ 1. ■