MATH 174, ANSWERS TO TEST 1

1.

p	q	$p \wedge q$	$p \vee q$	$\sim q$	$\sim q \rightarrow p$	$\sim q \rightarrow (p \wedge q)$	$\sim q \wedge \big(\sim q \rightarrow (p \wedge q)\big)$
T	T	T	T	F	T	T	F
T	F	F	T	T	T	F	F
F	T	F	T	F	T	T	F
F	F	F	F	T	F	F	F

2. $\sim q \rightarrow (p \wedge q)$

3. $p \vee q$ and $\sim q \rightarrow p$

4. None.

5. $\sim q \wedge \big(\sim q \rightarrow (p \wedge q)\big)$

6. $\sim p \wedge \sim q$

7. $q \rightarrow p$

8. $\sim q \rightarrow \sim p$

9. the contrapositive

10. The first two arguments are invalid and the last one is valid.

11. $\forall x \in \mathbb{R}, x \leq 1$ or $x > 3$

12. $\exists n \in \mathbb{Z}$ such that $n \geq 4$, n is even, and n is not the sum of two primes

13. Observe that

$$4 = 2 + 2, \qquad 6 = 3 + 3, \qquad 8 = 3 + 5, \qquad 10 = 5 + 5, \qquad \text{and} \qquad 12 = 5 + 7.$$

This shows by the method of exhaustion that for all $n \in \{4, 6, 8, 10, 12\}$, n is the sum of two primes.

14. Since $2^5 = 32$ and $5^2 = 25$, we see that $2^5 > 5^2$. Thus, we have shown that there exists an integer $n > 2$ (namely, one can take $n = 5$) such that $2^n > n^2$.

15. 2, 5, and 11

16. 3, 5, and 9

17. No. To see this, we need only observe that $221 = 13 \times 17$. (One should check to see if 221 has a prime divisor $\leq \sqrt{221}$. This means one should check to see if 221 is divisible by one of the primes 2, 3, 5, 7, 11, or 13.)

18. $25 \bmod 7 = 4$, $-33 \bmod 14 = 9$, and $3029282726252423222120191817161514131211109876543210 \bmod 4 = 2$

19. $\lceil 37.95 \rceil = 38$ and $\lfloor -37.95 \rfloor = -38$

20. Assume $\boxed{\sqrt{2}}$ is rational. Then there exist $\boxed{\text{integers}}$ a and b with $b \neq 0$, with $\sqrt{2} = a/b$, and with a/b reduced (so that a and b have no common prime factors). Since $\sqrt{2} = a/b$, we obtain

$$b\sqrt{2} = a \qquad \text{so that} \qquad 2b^2 = a^2.$$

We deduce that $\boxed{a}$ is even. Therefore, there is an integer k such that $\boxed{a = 2k}$. Substituting this into $2b^2 = a^2$, we obtain $\boxed{2b^2 = (2k)^2 = 4k^2}$ so that $b^2 = \boxed{2k^2}$. We deduce that $\boxed{b}$ is even. This is a contradiction since $\boxed{a/b \text{ is reduced and } a \text{ and } b \text{ are even}}$. Therefore, our assumption is wrong and $\sqrt{2}$ is $\boxed{\text{irrational}}$. ■