
MATH 174: TEST 1

Name _____

Instructions: Put your name in the space provided above. Make sure that your test has six different pages including one blank page. Work each problem below and show ALL of your work. Put all answers in the spaces provided. Do NOT use a calculator.

Point Values: The point values are indicated to the left of each problem.

- (6) 1. Complete the truth table below.

p	q	$p \wedge q$	$p \vee q$	$\sim q$	$\sim q \rightarrow p$	$\sim q \rightarrow (p \wedge q)$	$\sim q \wedge (\sim q \rightarrow (p \wedge q))$
T	T	T	T	F			
T	F	F	T	T			
F	T	F	T	F			
F	F	F	F	T			

- (4) 2. Which one of the statement forms $p \wedge q$, $p \vee q$, $\sim q$, $\sim q \rightarrow p$, $\sim q \rightarrow (p \wedge q)$, and $\sim q \wedge (\sim q \rightarrow (p \wedge q))$ is equivalent to q ? (Note: These statement forms are in the truth table in Problem 1.)

Answer:

- (4) 3. Which two of the statement forms $p \wedge q$, $p \vee q$, $\sim q$, $\sim q \rightarrow p$, $\sim q \rightarrow (p \wedge q)$, and $\sim q \wedge (\sim q \rightarrow (p \wedge q))$ are equivalent to each other? (Note: These statement forms are in the truth table in Problem 1.)

Answers:

1.

2.

(put one form in each box)

- (4) 4. Which if any of the statement forms $p \wedge q$, $p \vee q$, $\sim q$, $\sim q \rightarrow p$, $\sim q \rightarrow (p \wedge q)$, and $\sim q \wedge (\sim q \rightarrow (p \wedge q))$ is a tautology? Put “NONE” for the answer if none of these forms is a tautology. (Note: These statement forms are in the truth table in Problem 1.)

Answer:

- (4) 5. Which if any of the statement forms $p \wedge q$, $p \vee q$, $\sim q$, $\sim q \rightarrow p$, $\sim q \rightarrow (p \wedge q)$, and $\sim q \wedge (\sim q \rightarrow (p \wedge q))$ is a contradiction? Put “NONE” for the answer if none of these forms is a contradiction. (Note: These statement forms are in the truth table in Problem 1.)

Answer:

- (4) 6. According to De Morgan's Law, $\sim (p \vee q)$ is equivalent to what?

Answer:

- (4) 7. What is the converse of “if p then q ”? Use either words or a statement form.

Answer:

- (4) 8. What is the contrapositive of “if p then q ”? Use either words or a statement form.

Answer:

- (4) 9. Which one of the converse, the inverse, the negation, and the contrapositive of “if p then q ” is equivalent to “if p then q ”?

Answer:

- (9) 10. Determine whether the following arguments are valid or invalid.

If I will go to school,
then I will learn a lot.
I will learn a lot.
 \therefore I will go to school.

Answer:
(write "valid" or write "invalid")

For every martian boy, if he lives
on Venus, then he is green.
Joho is a green martian boy.
 \therefore Joho lives on Venus.

Answer:
(write "valid" or write "invalid")

No student in this class
knows Bill Satchaquaw.
Jill is a student who
knows Bill Satchaquaw.
 \therefore Jill is not in this class.

Answer:
(write "valid" or write "invalid")

- (4) 11. Negate the following:

$$\exists x \in \mathbb{R} \text{ such that } x > 1 \text{ and } x \leq 3$$

Answer:

- (4) 12. Negate the following:

$$\forall n \in \mathbb{Z}, \text{ if } n \geq 4 \text{ and } n \text{ is even, then } n \text{ is the sum of two primes}$$

Answer:

- (4) 13. Prove that for all $n \in \{4, 6, 8, 10, 12\}$, n is the sum of two primes. The two primes may be equal (they may be the same prime). Use complete English sentences.

Put Proof Here:

- (4) 14. Prove that there exists an integer $n > 2$ such that $2^n > n^2$. Use complete English sentences (one sentence is fine here).

Put Proof Here:

- (4) 15. What are the prime divisors of 440?

Answer:

- (4) 16. Which of the numbers 3, 4, 5, and 9 are divisors of 9876543210? Your answer should be more than one number.

Answer:

(4) 17. Is 221 a prime number?

Answer:
(write "Yes" or write "No")

(9) 18. Calculate each of the following:

$$25 \bmod 7 = \boxed{}$$

$$-33 \bmod 14 = \boxed{}$$

$$3029282726252423222120191817161514131211109876543210 \bmod 4 = \boxed{}$$

(Note: We discussed how to get the remainder quickly when dividing by 4.)

(6) 19. Calculate each of the following:

$$\lceil 37.95 \rceil = \boxed{}$$

$$\lfloor -37.95 \rfloor = \boxed{}$$

(10) 20. Complete the following proof that $\sqrt{2}$ is irrational.

Assume is rational. Then there exist a and b with $b \neq 0$, with $\sqrt{2} = a/b$, and with a/b reduced (so that a and b have no common prime factors). Since $\sqrt{2} = a/b$, we obtain

$$b\sqrt{2} = a \quad \text{so that} \quad 2b^2 = a^2.$$

We deduce that is even. Therefore, there is an integer k such that . Substituting this into $2b^2 = a^2$, we obtain so that $b^2 = \text{input}$. We deduce that is even. This is a contradiction since

.

Therefore, our assumption is wrong and $\sqrt{2}$ is . ■