MATH 174, PRACTICE PROBLEMS FOR TEST 2

1. Complete the summation notation below.

$$1+3+3^2+3^3+\cdots+3^{99}+3^{100} = \sum_{k=1}^{\boxed{}} \boxed{}$$

2. Complete the summation notation below.

$$1-3+3^2-3^3+\cdots-3^{99}+3^{100} = \sum_{k=1}^{\boxed{}} \boxed{}$$

3. Evaluate
$$\sum_{k=1}^{20}\left(\frac{1}{k+1}-\frac{1}{k+2}\right).$$

4. What is the value of the sum
$$20+22+24+26+\cdots+98+100\,?$$

5. What is the value of
$$2^{10}-2^8+2^6-2^4+2^2-1\,?$$
Express your answer in the form $\frac{1}{a}\big(2^b-c\big)$ for some integers a, b, and c.

6. Which if any of $(A\cap B)\cup(A\cap C)$ and $(A\cup B)\cap(A\cup C)$ is the same as $A\cap(B\cup C)$?

7. What are DeMorgan's Laws for sets?

8. True or False (justify your answer). If $A\cup C = B\cup C$, then $A = B$.

9. True or False (justify your answer). If $A\cap C = B\cap C$, then $A = B$.

10. If $A=\{a,b,c\}$, $B=\{a,c,e,f\}$, $C=\{c,d,f,g\}$, and U is a universal set containing A, B, and C, then what are the following: (a) $\big(A\cap B\big)-C$ (b) $\big(B\cup C\big)\cap\big(A-C^c\big)$

11. If $A=\{1,2\}$, then what is $A\times A$?

12. Do the sets $A=\{n\in\mathbb{Z}: n \bmod 2 = 1\}$, $B=\{n\in\mathbb{Z}: n \bmod 4 = 0\}$, and $C=\{n\in\mathbb{Z}: n \bmod 4 = 2\}$ partition the set $\mathbb{Z}$?

13. What is the power set of $\{a,b,c\}$? (Indicate its elements.)

14. Kelly reads a chapter of a book beginning at the top of page 80 and ending at the bottom of page 100. How many pages has Kelly read in this chapter?

15. What is the 20^{th} element in the array
$$A[18], A[20], A[22], \ldots, A[198], A[200]\,?$$

16. How many different ways can the letters in THREE be arranged in a row?

17. How many different ways can the letters in THREE be arranged in a row if the two E's are *not* to be next to each other?

18. There are 5 women and 4 men wanting to form a committee using 2 of the women and 2 of the men. How many different committees are possible? Simplify your answer.

19. If the product $(3x-2)^{11}(x-2)^{15}(12x^3-10x^2+2x+5)$ is expanded, the result is a 29^{th} degree polynomial. In other words, the expanded product is of the form
$$a_{29}x^{29}+a_{28}x^{28}+\cdots+a_2x^2+a_1x+a_0.$$
What is the sum of the coefficients $a_{29}, a_{28}, \ldots, a_1$ and a_0?

20. The 10^{th} row of Pascal's triangle is

$$1 \quad 10 \quad 45 \quad 120 \quad 210 \quad 252 \quad 210 \quad 120 \quad 45 \quad 10 \quad 1.$$

What is the 11^{th} row?

21. What is the coefficient of x^5 in the expansion of $(2x-1)^8$?

22. A chessboard is an 8×8 board. It contains 64 small squares, but it also contains some larger squares.
   (a) How many 2×2 squares are on a chessboard?
   (b) How many 3×3 squares are on a chessboard?
   (c) How many 7×7 squares are on a chessboard?
   (d) How many total squares are on a chessboard? Express your answer using summation notation and evaluate the sum.

23. Complete the following proof that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{n-1}} + \frac{1}{\sqrt{n}} \ \geq \sqrt{n}.$$

Let $P(n)$ be the statement that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{n-1}} + \frac{1}{\sqrt{n}} \ \geq \boxed{}.$$

We prove that $P(n)$ is true for every positive integer n by using $\boxed{}$. We first show that $\boxed{}$ is true. Since $\sqrt{1} = 1$, we see that $1/\sqrt{1} \geq \sqrt{1}$. Thus, $\boxed{}$ is in fact true. Next, we suppose that k is an integer ≥ 1 such that $\boxed{}$ is true. This is called our induction hypothesis. Thus, our induction hypothesis is asserting that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{k-1}} + \frac{1}{\sqrt{k}} \ \geq \boxed{}.$$

From the induction hypothesis, we obtain that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{k-1}} + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} \geq \sqrt{k} + \frac{1}{\sqrt{k+1}}.$$

Observe that $\sqrt{k(k+1)} > \sqrt{k^2} = k$ so that

$$\sqrt{k(k+1)} + 1 > k + 1.$$

Dividing by $\sqrt{k+1}$, we deduce that

$$\boxed{} + \frac{1}{\sqrt{k+1}} > \boxed{}.$$

We deduce that

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{k-1}} + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} \geq \boxed{}.$$

We have shown that if $P(k)$ is true, then $\boxed{}$ is true. This completes the induction argument. Therefore, $P(n)$ is true for every integer $n \geq 1$. ■