MATH 174, PRACTICE PROBLEMS FOR TEST 1

1. Complete the truth table below.

p	q	$p \wedge q$	$\sim p$	$\sim q$	$p \vee \sim q$	$\sim p \vee q$	$\sim p \vee (p \wedge q)$	$\sim q \vee (p \wedge q)$	$(p \vee \sim q) \vee (\sim p \vee q)$
T	T								
T	F								
F	T								
F	F								

2. Which if any of $p \vee \sim q$, $\sim p \vee q$, $\sim p \vee (p \wedge q)$, $\sim q \vee (p \wedge q)$, and $(p \vee \sim q) \vee (\sim p \vee q)$ are equivalent?

3. Which if any of $p \vee \sim q$, $\sim p \vee q$, $\sim p \vee (p \wedge q)$, $\sim q \vee (p \wedge q)$, and $(p \vee \sim q) \vee (\sim p \vee q)$ are tautologies?

4. Which if any of $p \vee \sim q$, $\sim p \vee q$, $\sim p \vee (p \wedge q)$, $\sim q \vee (p \wedge q)$, and $(p \vee \sim q) \vee (\sim p \vee q)$ are contradictions?

5. According to De Morgan's Law, $\sim (p \vee q)$ is equivalent to what?

6. According to De Morgan's Law, $\sim (p \wedge q)$ is equivalent to what?

7. Why is it that any collection of 17 or more statement forms in the variables p and q must contain at least two forms that are equivalent?

8. What is the converse of "if p then q"?

9. What is the inverse of "if p then q"?

10. What is the negation of "if p then q"?

11. What is the contrapositive of "if p then q"?

12. Which of the converse, the inverse, the negation, and the contrapositive of "if p then q" is equivalent to "if p then q"?

13. Complete the truth table below.

p	q	$p \wedge q$	$\sim q$	$\sim q \to p \wedge q$
T	T			
T	F			
F	T			
F	F			

14. Which of the following arguments are valid?

If I go to the moon, then I will eat cheese. I will go to the moon. $\therefore$ I will eat cheese.	If I go to the moon, then I will eat cheese. I will not eat cheese. $\therefore$ I will not go to the moon.	For every student in Math 174, if the student shows up to class, then the student will do well. Bill is a student in Math 174 but he does not show up to class. $\therefore$ Bill will not do well.	No student in this class is getting an A. John is a student that is not in this class. $\therefore$ John is getting an A.

15. Negate $\forall x \in D, P(x)$ (where $P(x)$ is a predicate).

16. Negate $\exists x \in D$ such that $P(x)$ (where $P(x)$ is a predicate).

17. Negate $\forall$ positive integers n, $\exists$ integers $a, b, c,$ and d such that $n = a^2 + b^2 + c^2 + d^2$.

18. Negate $\forall a \in \mathbb{Q}, \forall b \in \mathbb{Q}, \exists c \in \mathbb{Q}$ such that $a < c < b$.

19. Let $D = \{3, 4, 5, 6, 7, 8\}$ be the domain for x. Let $P(x)$ be the predicate "x is divisible by 2" and $Q(x)$ be the predicate "x is a prime". Which of the following is true?

$$P(x) \implies Q(x), \quad \sim Q(x) \implies P(x), \quad P(x) \iff Q(x), \quad P(x) \iff \sim Q(x).$$

20. Prove that $\forall x \in \{0, 1, 2, 3, 4\}$, there does not exist an integer y such that $8x + 13y = 1$.

21. Prove that there exist integers x and y such that $8x + 13y = 1$.

22. What are the divisors of 12?

23. What are the prime factors of 195?

24. Completely factor 80.

25. Is 987654321 divisible by 3?

26. What day of the week will it be 200 days from now?

27. Is 241 prime? Justify your answer.

28. Calculate each of the following:

$$73 \bmod 5, \qquad -73 \bmod 5, \qquad 29 \bmod 4, \qquad -29 \bmod 4$$

29. Calculate each of the following:
$$\lfloor 3.6 \rfloor, \qquad \lceil 3.6 \rceil, \qquad \lfloor -1.9 \rfloor, \qquad \lceil -1.9 \rceil$$

30. Complete the following proof that $\sqrt{2}$ is irrational.

Assume $\sqrt{2}$ is rational. Then there exist [   ] a and b with $b \neq 0$, with $\sqrt{2} = a/b$, and with a/b reduced (so that a and b have no common prime factors). Since $\sqrt{2} = a/b$, we obtain

$$b\sqrt{2} = a \qquad \text{so that} \qquad 2b^2 = a^2.$$

We deduce that a is even. Therefore, there is an integer k such that [   ]. Substituting this into $2b^2 = a^2$, we obtain [   ] so that $b^2 =$ [   ]. We deduce that [   ] is even. This is a contradiction since [   ]. Therefore, our assumption is wrong and $\sqrt{2}$ is irrational. ∎