MATH 174, PRACTICE PROBLEMS FOR *After* TEST 2

1. Let u_n be a sequence defined recursively by $u_0 = -1/2$, $u_1 = 0$, and $u_{n+1} = 6u_n - 8u_{n-1}$ for all $n \geq 1$.

   (a) What are the values of u_2, u_3, and u_4?
   (b) Find an explicit formula for u_n.

2. Suppose u_n is a sequence defined by $u_n = 6^n - 7 \times 2^n$ so that $u_0 = -6$, $u_1 = -8$, and $u_2 = 8$. Then u_n satisfies which one of the following recursion relations for all $n \geq 1$?

   (a) $u_{n+1} = 2u_n - 4u_{n-1}$ (b) $u_{n+1} = 5u_n - 8u_{n-1}$ (c) $u_{n+1} = 8u_n - 12u_{n-1}$ (d) $u_{n+1} = 11u_n - 16u_{n-1}$

3. Suppose u_n is a sequence defined by $u_n = 2^n - 3 \times (-1)^n$ so that $u_0 = -2$, $u_1 = 5$, and $u_2 = 1$. Determine a recursion relation that u_n satisfies for all $n \geq 1$.

4. Suppose $f(x) = O(g(x))$ so that there are positive M and x_0 such that $|f(x)| \leq M|g(x)|$ for every $x \geq x_0$. Suppose also $g(x) = O(h(x))$ so that there are positive M' and x_0' such that $|g(x)| \leq M'|h(x)|$ for every $x \geq x_0'$. Explain why $f(x) = O(h(x))$. In other words, why are there M'' and x_0'' such that $|f(x)| \leq M''|h(x)|$ for every $x \geq x_0''$?

5. Which of the following (possibly more than one) is $O(x^2)$? Give brief justifications of your answers (for each of (a), (b), (c), (d), and (e), you should explain why it is or is not $O(x^2)$).

   (a) x (b) x^3 (c) $(x-1)^2$ (d) $(x+1)^2$ (e) $(\log_2 x)^{24}$

6. Which of the following (possibly more than one) is $O(n^5)$? Give brief justifications of your answers (for each of (a), (b), (c), and (d), you should explain why it is or is not $O(n^5)$).

   (a) $\sum_{k=1}^{n} k^4$ (b) $\sum_{k=1}^{n} k^5$ (c) $\sum_{k=1}^{n} k(k+1)(k+2)(k+3)$ (d) $\sum_{k=1}^{n} 2^k$

7. For the graph below, determine the degrees of the vertices A, B, and C.

8. Denote a path by a sequence of vertices. For example, $DECDA$ would be the path starting at D, and then going to E, and then C, and then back to D, and then ending at A. Write a sequence of vertices that represents an Euler circuit for the graph below.

9. Is the complete graph on six vertices (denoted K_6) planar? Why or why not?